Influence of Mechanical Vibrations on the Field Quality Measurements of LHC Interaction Region Quadrupole Magnets

J. DiMarco[(1)], H. Glass[(1)], J. Krzywinski[(2)], P. Schlabach[(1)], C. Sylvester[(1)], J.C. Tompkins[(1)]
(1) Fermilab, Batavia, IL, USA, (2) Inst. of Physics, Polish Academy of Sciences, Warsaw, Poland.

***Abstract* - The high gradient quadrupole magnets being developed by the US-LHC Accelerator Project for the LHC Interaction Regions have stringent field quality requirements. The field quality of these magnets will be measured using a rotating coil system presently under development. Mechanical vibrations of the coil during field quality measurements are of great concern because such vibrations can introduce systematic errors in measurement results. This paper presents calculations of the expected influence of vibrations on field quality measurements and a technique to measure vibrations present in data acquired with standard "tangential-style" probes. Measured vibrations are reported and compared to simulations. Limits on systematic errors in multipole measurements are discussed along with implications for probe and measurement system design.**

I. INTRODUCTION

Field requirements for the low-β insertion quadrupoles can be very demanding because their strong focusing causes the beta function (and therefore the beam radius) to be considerably larger than in the arc magnets. The requirements on the accuracy of magnetic field measurements of these magnets is accordingly quite high. Several potential sources of error can be neglected: 1) random noise effects are reduced through statistics; 2) electrical noise effects are negligible given sufficient signal; and 3) electronics and encoders non-linearity are insignificant when the appropriate bucking is applied [1]. Relative probe manufacturing precision, dr/r, of the windings influence relative harmonics measurements accuracy, dc_n/c_n, according to

$$\frac{dc_n}{c_n} \approx n\frac{dr}{r}$$

(n is harmonic number and r the radius of the coil), and since manufacturing precisions of ~50 μm are common, the relative measurement accuracy for a 1 cm probe is only a few percent even for n=14; this is typically more than sufficient.

Erratic movements of the coil, however, warrant further attention since these limited high-order multipole measurements in HERA arc dipole and quadrupole magnets [2]. In this paper, the influence of irregularities in coil motion on the measurement accuracy of field harmonics is examined. First, expressions for fake multipoles caused by vibrations are determined, then a method to measure vibrations is given, and finally data from HGQ model magnets is discussed.

II. Effects of Coil Motion on Measured Harmonics

To examine the effects of coil motion, we begin with a single-wire radial probe (i.e. a probe which has one wire at a radius r with respect to it rotation axis, returning along r=0), and note that any rotating probe geometry can be constructed by superimposing a collection of such individual radial coils. We consider motion errors that stem from translations of the probe transverse to its axis of rotation, as well as angular errors caused by torsional vibrations.

(a) Transverse Probe Motion Errors

The coordinate system for a rotation of the probe in the magnet is defined as in Figure 1.

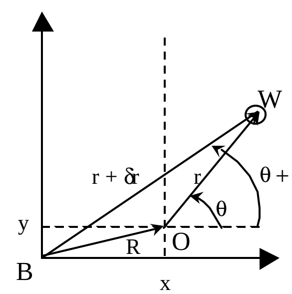

Fig.1. Coordinate definitions for a single wire, *W*, moving about its rotation axis, *B*, in a magnet with field center defined by *O*.

The average location of the probe center is denoted in the figure by point *B*, shown at an arbitrary motion displacement with respect to the magnet center, position *O*. The tangential component of the field is defined in the standard way,

$$B_\theta(r,\theta) = \Re\left\{ g r_0 \sum_{n=1}^{\infty} c_n \left(\frac{r}{r_0}\right)^{n-1} e^{in\theta} \right\} \tag{1}$$

where $c_n = b_n + ia_n$ are the complex harmonic coefficients, r_0 the reference radius, g the quadrupole field gradient, and $\Re$ denotes the real part of the complex quantity. The magnetic flux of the imperfect rotation as seen by the wire can then be expressed as

$$\Phi(r,\theta) = \frac{L}{n} g r_0^2 \Re\left\{ \sum_{n=1}^{\infty} \frac{1}{n} c_n \left[\left(\frac{r+\delta r(\theta)}{r_0}\right)^n e^{in(\theta+\delta\theta(\theta))} - \left(\frac{R(\theta)}{r_0}\right)^n e^{in\phi(\theta)} \right] \right\} \tag{2}$$

where L is the wire length, R is given by $R = \sqrt{x^2 + y^2}$, and $\phi = \arctan\frac{y}{x}$. When $\frac{\delta r}{r}$ and $\delta\theta$ are small, they can be expressed as translations in x and y according to

$$
\begin{aligned}
\delta\theta &\cong \frac{-x}{r}\sin\theta + \frac{y}{r}\cos\theta \\
\frac{\delta r}{r} &\cong \frac{x}{r}\cos\theta + \frac{y}{r}\sin\theta
\end{aligned}
\tag{3}
$$

Expanding Eqn. (2) (keeping terms linear in $\frac{\delta r}{r}$ and $\delta\theta$) and applying the relations of Eqn. (3), gives

$$
\Phi(r,\theta) = \frac{L}{n} g r_0^2 \Re\left\{ \sum_{n=1}^{\infty} \frac{1}{n} c_n \left(\frac{r}{r_0}\right)^n e^{in\theta} \left[1 + n\left(\frac{x(\theta)+iy(\theta)}{r_0} \right) e^{-i\theta} \right] \right\} \tag{4}
$$

Since systematic multipole measurement errors arise from motion errors which are a periodic function of θ (as these cannot be averaged out), we can express these movements of the coil axis in terms of the Fourier series

$$
x(\theta) = \Re\left\{ \sum_{n=1}^{\infty} x_k e^{ik\theta} \right\}, \quad y(\theta) = \Re\left\{ \sum_{n=1}^{\infty} y_k e^{ik\theta} \right\} \tag{5}
$$

where for each vibration order k, the x_k, y_k amplitudes are complex (having amplitude as well as some phase with respect to the rotation angle θ). Inserting Eqn. (5) into Eqn. (4) and defining $z_k \equiv \frac{x_k + iy_k}{r}$, the following expressions for false harmonics generated by the transverse motion errors are obtained:

$$
\begin{aligned}
\delta b_n &= \frac{n}{2} \frac{\sum_{k=1}^{\infty} \Re(z_k) \left[b_{|n-k|+1} \left(\frac{r}{r_0}\right)^{|n-k|+1} + b_{n+k+1} \left(\frac{r}{r_0}\right)^{n+k+1} \right]}{\left(\frac{r}{r_0}\right)^n} \\
&+ \frac{n}{2} \frac{\sum_{k=1}^{\infty} \Im(z_k) \left[-a_{|n-k|+1} \left(\frac{r}{r_0}\right)^{|n-k|+1} \operatorname{sgn}(n-k+1) + a_{n+k+1} \left(\frac{r}{r_0}\right)^{n+k+1} \right]}{\left(\frac{r}{r_0}\right)^n} \\
\delta a_n &= \frac{n}{2} \frac{\sum_{k=1}^{\infty} \Im(z_k) \left[-b_{|n-k|+1} \left(\frac{r}{r_0}\right)^{|n-k|+1} + b_{n+k+1} \left(\frac{r}{r_0}\right)^{n+k+1} \right]}{\left(\frac{r}{r_0}\right)^n} \\
&+ \frac{n}{2} \frac{\sum_{k=1}^{\infty} \Re(z_k) \left[a_{|n-k|+1} \left(\frac{r}{r_0}\right)^{|n-k|+1} \operatorname{sgn}(n-k+1) + a_{n+k+1} \left(\frac{r}{r_0}\right)^{n+k+1} \right]}{\left(\frac{r}{r_0}\right)^n}
\end{aligned}
\tag{6}
$$

(b) Angular Probe Motion Errors

The derivation of multipole measurement errors for angular errors follows closely that for transverse motion errors. The measurement errors are given by

$$
\begin{aligned}
\delta b_n &= \frac{n}{2} \frac{\sum_{k=1}^{\infty} \eta_k \left[-b_{|n-k|} \left(\frac{r}{r_0}\right)^{|n-k|} \operatorname{sgn}(n-k) + b_{n+k} \left(\frac{r}{r_0}\right)^{n+k} \right]}{\left(\frac{r}{r_0}\right)^n} \\
&- \frac{n}{2} \frac{\sum_{k=1}^{\infty} \varepsilon_k \left[a_{|n-k|} \left(\frac{r}{r_0}\right)^{|n-k|} + a_{n+k} \left(\frac{r}{r_0}\right)^{n+k} \right]}{\left(\frac{r}{r_0}\right)^n} \\
\delta a_n &= \frac{n}{2} \frac{\sum_{k=1}^{\infty} \eta_k \left[-a_{|n-k|} \left(\frac{r}{r_0}\right)^{|n-k|} + a_{n+k} \left(\frac{r}{r_0}\right)^{n+k} \right]}{\left(\frac{r}{r_0}\right)^n} \\
&- \frac{n}{2} \frac{\sum_{k=1}^{\infty} \varepsilon_k \left[-b_{|n-k|} \left(\frac{r}{r_0}\right)^{|n-k|} \operatorname{sgn}(n-k) + b_{n+k} \left(\frac{r}{r_0}\right)^{n+k} \right]}{\left(\frac{r}{r_0}\right)^n}
\end{aligned}
\tag{8}
$$

where the Fourier components of the torsional vibrations are expressed as

$$
\delta\theta = \Re\left\{ \sum_{n=1}^{\infty} \delta\theta_k e^{ik\theta} \right\}, \quad \delta\theta_k = \eta_k + i\varepsilon_k
$$

Eqns (6) and (8) above were checked against the results of probe motion simulation and found to be in nearly exact agreement. These equations show multipole errors are strong functions of r_0/r for the higher orders, implying that probes with as large a radius as practicable should be used.

III. Measuring Vibrations in Rotating Coil Systems

A schematic of the cross-section for a typical bucked tangential rotating probe is shown in Figure 2.

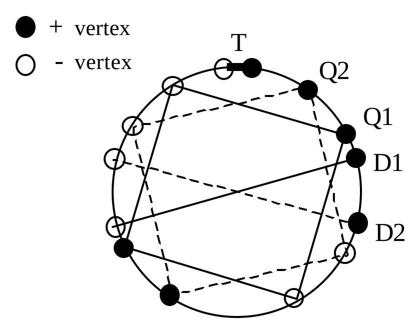

Fig.2. Schematic cross-section of a bucked tangential winding probe.

This probe design uses the tangential winding for measurement of higher order harmonics, while the bucking windings measure and remove the fundamental field components. Two bucking windings are present for each order of harmonic to be bucked out (in this case, two dipole buck and two quadrupole buck windings). This allows combination of buck windings to match the phase of the tangential winding to as high degree as possible.

(a) Transverse Probe Vibrations

From Eqn. (4), we can express the quadrupole field for which transverse vibration errors of the probe are present as

$$\Phi(q) = \frac{L}{n} g r_0^2 \Re\left\{ \sum_{n=1}^{\infty} \frac{1}{n} c_n \left(\frac{r}{r_0}\right)^n e^{inq} \left[1 + n\left(\frac{x(q)+iy(q)}{r_0}\right) e^{-iq} \right] \right\}$$

Since the error term has dipole periodicity, the dipole windings are sensitive to these variations in flux. Expressing the flux measured in the two dipole windings (D_1 and D_2) with cosine functions separated in phase by g (the angle between the two windings) plus the phase error from the vibrations (given approximately by $x(q)/r$), gives

$$\begin{aligned} \Phi_{D1}(q) &= f_{D1} \cos\left(q + a + \frac{x(q)}{r} \right) \\ \Phi_{D2}(q) &= f_{D2} \cos\left(q + a + g + \frac{x(q)}{r} \right) \end{aligned} \qquad (10)$$

where a is the phase angle of D_1 with respect to q, and the f's are the measured flux amplitudes. Solving Eqn (10) for $x(q)$ small, gives the $x(q)/r$ vibration error function,

$$\frac{x(q)}{r} = \frac{\dfrac{\Phi_{D1}(q)}{f_{D1}} \cos(q + a + g) - \dfrac{\Phi_{D2}(q)}{f_{D2}} \cos(q + a)}{\sin(g)} \qquad (11)$$

A similar result for the $y(q)/r$ vibration function can be obtained by expressing the dipole buck windings in terms of sine functions, yielding

$$\frac{y(q)}{r} = \frac{\dfrac{\Phi_{D1}(q)}{f_{D1}} \sin(q + a + g) - \dfrac{\Phi_{D2}(q)}{f_{D2}} \sin(q + a)}{\sin(g)} \qquad (12)$$

(b) Torsional Probe Vibrations

For torsional vibrations, the quadrupole buck windings are used to determine the error function. The flux from the quadrupole windings (including terms for angle errors caused by torsional vibrations) can be expressed as

$$\begin{aligned} \Phi_{Q1}(q) &= f_{Q1} \cos(2q + a + 2dq(q)) \\ \Phi_{Q2}(q) &= f_{Q2} \cos(2q + a + 2g + 2dq(q)) \end{aligned}$$

from which the dq/q torsional vibration function can be determined as

$$dq(q) = \frac{\dfrac{\Phi_{Q1}(q)}{f_{Q1}} \cos(2q + a + 2g) - \dfrac{\Phi_{Q2}(q)}{f_{Q2}} \cos(2q + a)}{2\sin(2g)} \qquad (13)$$

(c) Results

Eqns. (11-13) can be used to measure deviations in the transverse or angular motion of the coil during probe rotation.

To verify to what extent the vibrations were measured correctly, simulated signals were generated using the vibration results as input. An example comparison of raw and simulated signals is shown in Figure 3.

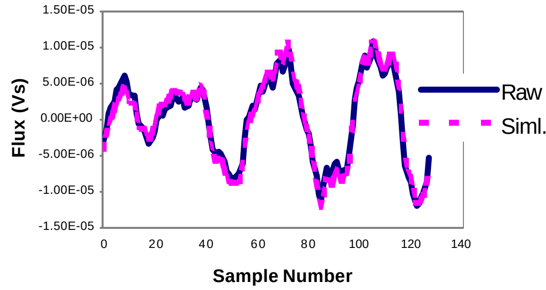

Fig.3. Comparison of raw dipole buck winding signal and signal generated from simulation with measured vibrations (note that the dipole and quadrupole orders have been suppressed for clarity).

The fairly good agreement between raw and simulated residuals indicates that the measured vibrations will provide reasonable estimates of multipoles errors. Typical amplitude spectra for transverse and torsional vibrations during VMTF measurements are given in Table I, along with the expected false harmonics in the tangential winding from this set of vibrations. The VMTF probe has a radius of r = 0.0196m.

TABLE I

TYPICAL VIBRATIONS FOR MEASUREMENTS AT VMTF AND THEIR SIMULATED ERRORS IN UNBUCKED TAN SIGNAL

	Vibration Amplitudes		Error
n	Lateral (m) / r (m)	Torsional (rad)	units @ 17mm
1	0.006618	0.000060	0.000
2	0.000137	0.000016	0.044
3	0.000652	0.000015	0.148
4	0.000044	0.000019	0.776
5	0.000025	0.000063	0.259
6	0.000033	0.000064	0.600
7	0.000020	0.000061	0.180
8	0.000018	0.000064	0.116
9	0.000015	0.000052	0.025
10	0.000013	0.000056	0.058
11	0.000010	0.000034	0.027
12	0.000008	0.000025	0.074

Note that some error in the vibration spectrum determination is expected as e.g. the n=4 torsion vibrations generate δb_2, which is indistinguishable from the main field and does not show up in the torsional analysis. Also, some multipole content appearing in the windings may originate from imperfections in probe manufacturing but would be interpreted as stemming from vibrations (e.g. the presence of δb_1 in the quad windings). Finally, other multipoles that the windings are sensitive to (e.g. b_6 in the quad buck windings) will be interpreted as stemming from vibrations.

IV. Measurements with Induced Vibrations

Table I, shows substantial multipole errors from vibrations are to be expected. However, bucking helps eliminate the sensitivity of the probe to the fundamental field, allowing a much greater tolerance to vibrations. Measurements were made at VMTF with and without induced vibrations to determine 1) whether multipole errors match simulated vibration errors, and 2) to what extent digital bucking subsequently removes vibration effects and allows accurate measurement of multipoles.

TABLE II
INDUCED VIBRATION SPECTRA FOR MEASUREMENTS AT VMTF

	Lateral (m) / r (m)			Torsional (radians)		
n	light	med	heavy	light	med	heavy
1	0.011695	0.011015	0.011173	0.000137	0.000131	0.000197
2	0.003526	0.003021	0.003325	0.000034	0.000060	0.000202
3	0.000916	0.000792	0.000893	0.000029	0.000047	0.000327
4	0.000703	0.000697	0.000820	0.000027	0.000058	0.000219
5	0.000160	0.000198	0.000389	0.000111	0.000639	0.002630
6	0.000247	0.000239	0.000275	0.000118	0.000276	0.000726
7	0.000115	0.000207	0.000221	0.000047	0.000202	0.000487
8	0.000136	0.000082	0.000115	0.000098	0.000227	0.000420
9	0.000130	0.000145	0.000144	0.000124	0.000210	0.000411
10	0.000073	0.000077	0.000121	0.000179	0.000562	0.000858
11	0.000084	0.000066	0.000069	0.000148	0.000229	0.000246
12	0.000045	0.000066	0.000086	0.000034	0.000093	0.000200

To induce torsional vibrations, a fixed shim was forced into contact with a pentagonal attachment to the probe shaft. Transverse vibrations were induced under the same arrangement but with some of the center support for the probe removed so that it was less constrained laterally. The lateral and torsional vibration spectra for these runs is summarized in Table II.

TABLE III
MULTIPOLE ERRORS FROM DATA WITH INDUCED VIBRATIONS: MEASUREMENTS, AND SIMULATION USING MEASURED VIBRATIONS
(UNITS AT $r_0 = 0.017$m)

	light		med		heavy	
n	meas	sim	meas	sim	meas	sim
1	0.064	0.070	0.036	0.065	0.182	0.159
2	1.901	1.574	0.920	1.903	3.624	4.150
3	15.009	16.330	8.063	14.677	42.238	36.976
4	3.336	3.899	0.899	2.828	3.894	6.879
5	3.472	1.637	2.385	1.452	3.409	2.755
6	1.547	0.563	2.291	2.105	2.409	5.758
7	1.930	0.708	1.464	4.003	8.522	13.618
8	1.487	1.062	3.305	1.453	5.020	4.201
9	0.994	0.436	0.901	0.794	1.261	0.936
10	0.241	0.523	0.279	0.834	1.149	0.744
11	0.719	0.435	1.164	1.450	1.473	1.598
12	0.762	0.595	1.992	1.885	2.691	2.438

TABLE IV
MULTIPOLE ERRORS FROM DATA WITH INDUCED VIBRATIONS: AFTER BUCKING TAN WINDING WITH QUADRUPOLE AND DIPOLE WINDINGS
(UNITS AT $r_0 = 0.017$m)

n	light	med	heavy
1	0.001	0.001	0.002
2	0.041	0.041	0.041
3	0.036	0.038	0.042
4	0.006	0.010	0.010
5	0.013	0.015	0.019
6	0.008	0.006	0.001
7	0.001	0.002	0.005
8	0.002	0.002	0.006
9	0.001	0.000	0.000
10	0.001	0.002	0.002
11	0.001	0.001	0.004
12	0.000	0.001	0.000

For each level of induced vibration, the actual multipole errors present in the unbucked tangential winding measurements are compared to the errors obtained through simulation using measured vibrations. These data are summarized in Table III and show the presence of very large multipole errors. Rough agreement is seen between actual measurements and the expectation using measured vibrations.

With bucking, the effects of vibration are greatly reduced, as summarized in Table IV. The results from Tables III and IV suggest that for the probe system at VMTF, we can rely on the digital bucking to reduce vibration effects by a factor of 100-1000 even under extreme vibration conditions. Applying reduction factors in this range to the multipole errors of Table I (no induced vibrations) shows that vibration effects for the VMTF data are small compared to measurement requirements [3].

V. CONCLUSIONS

Calculations for the effects of vibrations on an arbitrary rotating coil system have been presented. Also, a method has been presented whereby the lateral and torsional vibration errors in a measurement system can be measured *in situ*, given the availablity of two dipole and two quadrupole buck windings on a rotating coil probe. Calculations and measured vibrations in VMTF data (with and without externally induced vibrations) have been compared to simulations with good results.

Determining whether a rotating coil measurement system can achieve a given level of measurement accuracy can be accomplished by:

1) measuring or estimating the vibration environment the system will operate in
2) determining the resulting calculated or simulated multipole errors (δb_n, δa_n).
3) estimating the final errors from vibration after conservative inclusion of the effects of bucking dipole and quadrupole components

Finally, note that since errors in angular position can effectively be removed by measuring the torsional vibration error function and correcting for it, in principle it should be possible to make measurements in quadrupole magnets without angular encoders (indexing only the '0' position of each rotation). Furthermore, when small vibration magnitude coupled with bucking reduces vibration errors to levels that are not significant, multipoles obtained in such an encoderless system should be correct even without applying corrections for torsional vibrations.

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