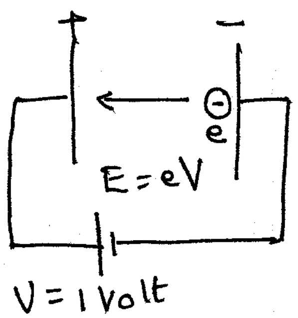
Units and Terminology in Accelerator physics

Like in any other discipline in physics a set of special physical units are in use in accelerator physics. These are primarily for convenience. Generally accelerator physics theory uses either

MKS system
or Gaussian cgs system } But neither is strictly adhered to.

Energy :-

Electron volt : Energy gained by an electron while being accelerated between two conducting plates at a potential difference of 1 - volt



The derivatives of the basic units are

$$1\ \text{keV} = 10^3\ \text{eV}$$
$$1\ \text{MeV} = 10^6\ \text{eV}$$
$$1\ \text{GeV} = 10^9\ \text{eV}$$
$$1\ \text{TeV} = 10^{12}\ \text{eV}$$
$$\vdots$$

To describe particle dynamics we use sometimes "momentum". This is because the Lorentz force is proportional to momentum.

other similar quantities are

1. Momentrem
2. Kinetic Energy
3. Total Energy.

At low energy (i.e., particle energy < rest mass energy) we have to keep difference between all these three. At high energies though the numerical values become similar we use

eV/c for momentrem

eV for Energy.

In case of Heavy ions we use kinetic energy/nu instead of total energy.

Beam Intensity :-

Beam current is measured in Amps or # of particles.

Circular Accelerators:

If βc = velocity of the particle

Z = charge multiplicity

f_{rev} = Revolution frequency

N = Total number of particles in the accelerator

Then intensity I is given by

$$I = eZ f_{rev} N$$

Transport line:

$$I = eZ\dot{N} \qquad \dot{N} = \text{flux of the particle}$$

The beam in Amp is given by

$$I(\text{Amp}) = I \cdot 1.602 \times 10^{-19}$$

For example in Tevatron :-

$Z = 1$, $f = 47000$, $N = 150 \times 10^9 \times 36 = 5.4 \times 10^{12}$

$$I(\text{Amp}) = 0.04 \text{ Amp}$$

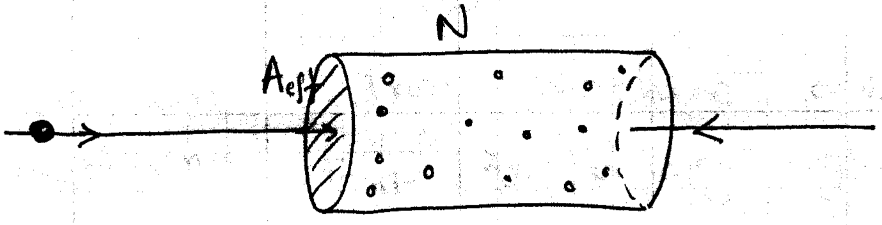
Linac :- since particles are continuously being accelerated the flux does not have meaning so we use peak current in a bunch.

Luminosity :-

In colliding beam physics we use "Luminosity" to represent beam intensity of both beams.
Luminosity is a measure of beam particle density in physical space and velocity space.

Suppose one bunch with N_+ - particles collides head-o with another bunch (with N_- particles) moving in opposit direction, then

Fraction of the area of the other bunch seen by one particle in the first bunch $= \dfrac{N_+ \sigma_{int}}{A_{eff}}$



Thus the number of interactions per passage of two such bunches

$$= N_- \frac{N_+ \sigma_{int}}{A_{eff}}$$

Suppose f is collision frequency, then

total number of interactions $= \dfrac{f N_+ N_- \sigma_{int}}{A_{eff}}$ —— (1a)

Then Luminosity $\mathcal{L}$ is defined by

$$\mathcal{L} = f \frac{N_+ N_-}{A_{eff}} \qquad \text{—— (1)}$$

For a Gaussian distribution it becomes

$$\mathcal{L} = f \frac{N_+ N_-}{4\pi\sigma^2} \qquad \text{—— (2)}$$

σ = rms width of the distn

Units of Luminosity : $cm^{-2}\ sec^{-1}$

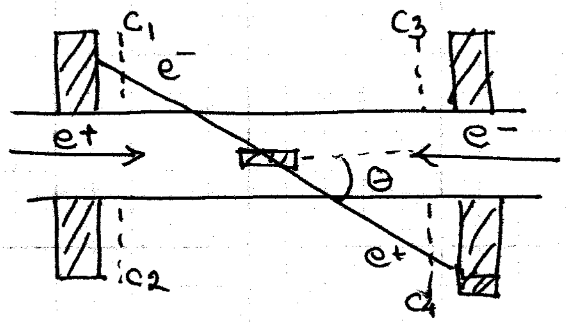
Very often luminosity is expressed in terms of "Inverse picobarn". This is done by integrating over time. Note that $1 \text{ barn} = 10^{-24}\ cm^2$

Measurement of Luminosity :-

Knowing every quantities in RHS of eqn 1 one can estimate luminosity. This is not a trivial job. Alternatively, by knowing a particular interaction rate we can estimate $\mathcal{L}$, e.g.,

$$\text{Total interaction rate} = \mathcal{L}\, \sigma_{int}$$

In an electron machine one can know "Bhabha-scattering" σ_{Int} to a very high precision.



$$\sigma_{int} = \int_{\theta_{min}}^{\theta_{max}} \frac{d\sigma_B}{d\Omega}\, d\Omega$$

$$\frac{d\sigma_B}{d\Omega} = \frac{\alpha^2}{8E^2} \frac{[2-\sin^2\theta][4-\sin^2\theta]}{\sin^4\theta}$$

By this method $\mathcal{L}$ can be measured to a few %.

In hadron machine $\sigma_{inelastic}$ is used (CDF)

The Van der Meer's Method:-

In the case of colliding beam with distributions ρ_1 and ρ_2, colliding at angle α, it can be shown that, luminosity is given by.

$$\mathcal{L} = \frac{I_1 I_2}{c e^2 h_{eff} \tan\alpha/2}$$

e = electron charge

I_1, I_2 = beam intensities

where

$$h_{eff} = \frac{\int_{-\infty}^{\infty} \rho_1(z)\, dz \int_{-\infty}^{\infty} \rho_2(z)\, dz}{\int_{-\infty}^{\infty} \rho_1(z)\, \rho(z)\, dz}$$

Vander Meer of CERN showed that h_{eff} can be measured very accurately by scanning one beam w.r.t. other vertically and measuring count rates.

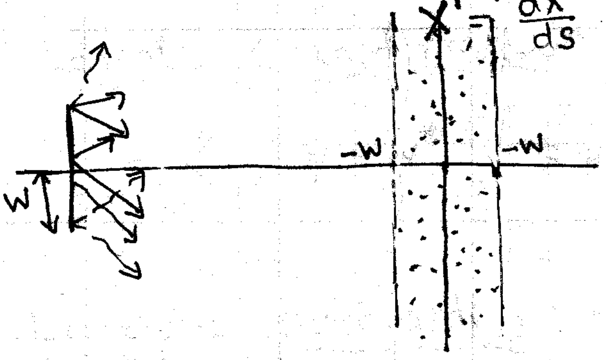
Emittance of Beam:-

Emittance is measure of transverse or longitudinal temperature of the beam. It is defined as total phase-space area occupied by the beam particles.

Emittance are expressed in units of π-mm-mradian
Longitudinal emittance are in units of eV-sec.

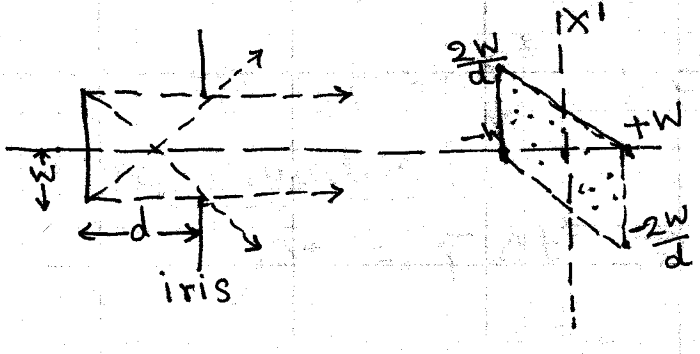
Conceptually Emittance can be understood as follows:-

Let us assume a diffuse source of particles of width W. The particles can emerge at any angle $0<\theta<90^\circ$ w.r.t. surface of the source. Then the phase space representation is in the domain $-W \leq x \leq W$ & $-\infty \leq x' \leq \infty$



Thus the particles are contained in a narrow strip.

In any accelerator a physical aperture will strictly define what is maximum acceptable phase space area. To illustrate this let us assume an "Iris" in the beamline.



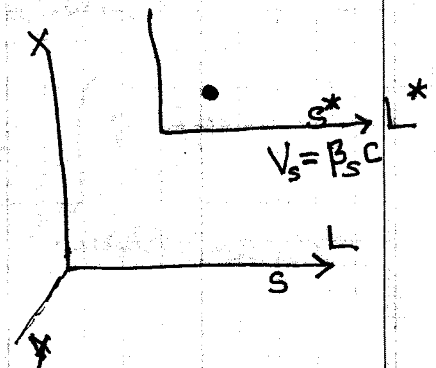
In this case beam has a specific area.

Relativistic Formalism

In high energy accelerators the particles will be accelerated to several times the particle rest mass energy from almost at rest. Therefore, relativistic mechanics is one of the prerequsit of accelerator physics

Beam dynamics is expressed generally in laboratory frame of reference.

Let L be lab-frame of reference. L^* be particle frame of reference with velocity $v_s = \beta_s c$



Lorentz transformation for coordinates are :-

$$x = x^*, \quad y = y^*, \quad s = \frac{s^* + \beta_s c t^*}{\sqrt{1-\beta_s^2}}, \quad ct = \frac{ct^* + \beta_s s^*}{\sqrt{1-\beta_s^2}}$$

Lorentz contraction :-

$$\Delta s = \gamma \Delta s^* \quad \text{where } \gamma = (1-\beta_s^2)^{-1/2}$$

$\Rightarrow V = \gamma V^*$; Volume scales linearly with γ

$\Rightarrow \rho = \frac{\rho^*}{\gamma}$; charge density scales as $\frac{1}{\gamma}$

Time Dialation :-

$\Delta t = \gamma \Delta t^* \Rightarrow$ time interval is shortest in particle frame of reference

Total Energy:-

Total energy of a particle is given by

$$E = \gamma m_o c^2$$
$$= \sqrt{p^2c^2 + m_o^2 c^4}$$

Kinetic Energy:-

$$E_{kin} = E - m_o c^2 = (\gamma - 1) m_o c^2$$

Kinetic Energy during Acceleration:-

Gain in kinetic Energy $\Delta E_{kin} = \int_{L_{accel.}} F.ds$

Particle Momentum:-

$$cp = \sqrt{E^2 - m_o^2 c^4}$$
$$= m_o c^2 \beta \gamma$$

- Differential forms:

$$dcp = \frac{m_o c^2}{\beta} d\gamma = \frac{dE}{\beta} = \frac{dE_{kin}}{\beta} = \gamma^3 m_o c^2 d\beta$$

$$\frac{dcp}{cp} = \frac{1}{\beta^2} \frac{d\gamma}{\gamma} = \gamma^2 \frac{d\beta}{\beta}$$

Electromagnetic Field:-

$$E_x^* = \gamma(E_x + \beta_s B_y) \qquad B_x^* = \gamma(B_x - \beta_s E_y)$$
$$E_y^* = \gamma(E_y - \beta_s B_x) \qquad B_y^* = \gamma(B_y + \beta_s E_x)$$
$$E_s^* = E_s \qquad B_z^* = B_z$$

$\Rightarrow$ Pure electric/magnetic field in Lab frame of reference will be combination of E and M field in particle frame!

Particle Collisions at High Energy :-

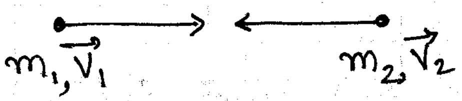
The total available energy from the collision depends on the kinematic parameters of colliding particles.

Center of mass coordinate system: This is a frame of reference where center of mass of the colliding particles is at rest.

Center of mass Energy:

$$E_{cm}^2 = (m_1\gamma_1 + m_2\gamma_2)^2 c^4 - (\gamma_1\beta_1 m_1 + \gamma_2\beta_2 m_2)^2 c^4$$

If $m_1 = m_2 = m$ $\quad V_2 = 0$



i.e., target at rest

$$E_{cm} = \sqrt{2(\gamma+1)}\; mc^2$$

∴ Available energy in a reaction is

$$E_{avail} = E_{cm} - 2mc^2 = \left\{\sqrt{2(\gamma+1)} - 2\right\} mc^2$$

Colliding beams with $m_1 = m_2 = m$, $V_1 = V_2 = \beta c$,

then

$$E_{cm} = 2\gamma mc^2 = 2E.$$

Maxwell's Equation

Accelerator physics is to a great extent the description of charged particle dynamics in the presence of external electromagnetic fields. Therefore Maxwell's Electromagnetic field equations are important ingredient in this discipline.

Gauss Law for E $\oint_S \vec{E}\cdot d\vec{A} = \frac{q}{\epsilon_0}$ ⇒ How electric charges give rise to electric field; field lines begin and end on charges.

Gauss Law for B $\oint_S \vec{B}\cdot d\vec{A} = 0$ ⇒ No magnetic charge; Magnetic field lines donot begin or end.

Faraday's law of induction $\oint_L \vec{E}\cdot d\vec{l} = -\frac{d\phi}{c\,dt}$ ⇒ Changing magnetic field induces electric field.

Ampere law $\oint_L \vec{B}\cdot d\vec{l} = \mu_0 I$ ⇒ A steady electric field gives rise to magnetic field.

ϵ_0 = permitivity constant = absolute dielectric constant

$= 8.854 \times 10^{-12}\ C^2/N\cdot m^2$

Coulomb Newton meter

μ_0 = Permiability constant

$= 4\pi \times 10^{-7}$ Tesla meter/amp

Lorentz Force :-

$$\vec{F} = q\vec{E} + \frac{q}{c}\vec{V} \times \vec{B}$$

change in momentum :

$$\Delta p = \int F \cdot dt$$

change in kinetic energy :

$$\Delta E_{kin} = \int F \cdot ds \quad ; \quad ds = \beta c \, dt$$

Then

$$\Delta E_{kin} = \beta c \, \Delta p$$

By substituting for F ; Lorentz force

$$\Delta E_{kin} = q \int E \cdot ds + \frac{q}{c} \int (\bar{V} \times \bar{B}) \, v \, dt$$