Types of Accelerators and Components of High Energy Accelerators Complex

The charged particle accelerators are broadly classified into two types.

1. linear accelerators

   examples:- Cascade accelerators, Van de Graaff, linac, RFQ

2. Circular accelerators

   Examples:- cyclotron, synchrocyclotron, Synchrotron, Betatron

Linear Accelerators:-

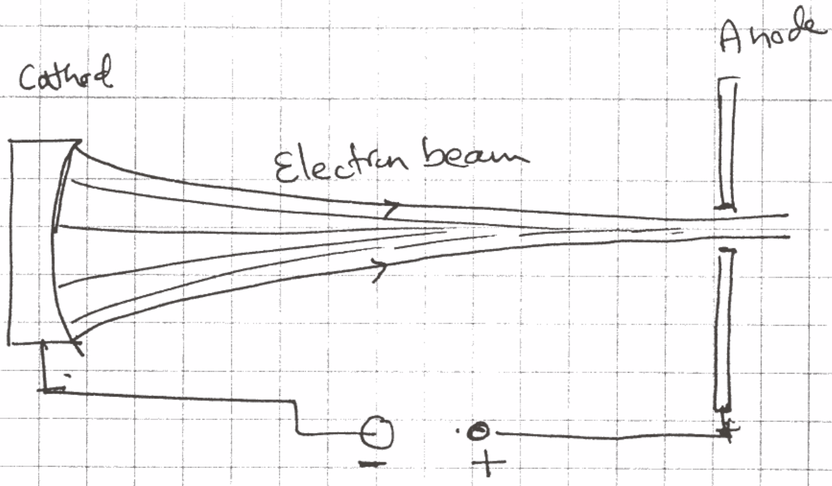
In linear accelerators the particles are accelerated by either electrostatic fields or by rf-fields.

Then according to eq^n (30a) and (30)

$$\Delta p = \int F\,dt = q\int E \cdot dt$$

$$\Delta E_{kin} = q\int \vec{E} \cdot d\vec{s}$$

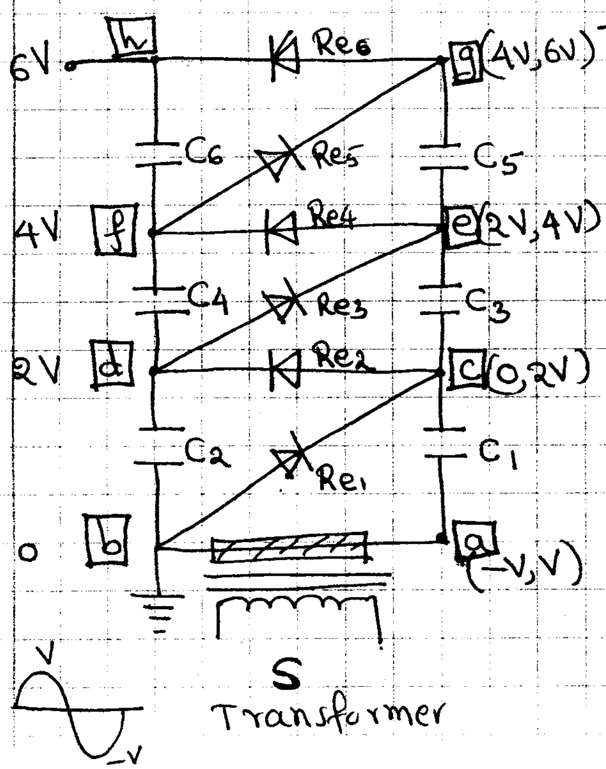
Electrostatic Accelerators:- Most simple arrangement used for two centuries is "glow discharge" tubes



thousand volts to
tens of thousands of volts

Cockcroft - Walton Accelerators or Cascade Accelerators

Historically cockcroft - Walton accelerators are the oldest types of accelerators and they are still in use as pre-accelerators (pre-injectors) in high-energy accelerator facilities.



This accelerator essentially consists of identical stages of capacitors "C" and rectifiers "Re" which gets power from an alternating current source "S" The peak voltage of the source is V. At point 'a' in the circuit the transformer sets up an alternating voltage which oscillates between (−V, +V).

The first stage rectifier Re_1 conducts when its anode is at +ve potential relative to the cathode. Thus in the +ve half-period the capacitor C_1 will be charged to Voltage "V". During −ve half period rectifier Re_1 will not conduct. However, Re_2 being opened will transfer half of the charges to C_2 During the next half period C_1 will be fully charged, while C_2 will give up half of what has gained previously to C_3. Thus in course of time all capacitor C1 to C6 will be charged fully and voltage between "b" to "h" will be 6V. The cascade generator shown above are "asymmetric" generator.

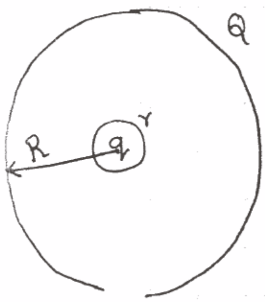
Thus if cascade generator have n identical stages then the total voltage out put will be

$$U_0 = 2nV \quad \text{--- (1)}$$

The Fermilab Cockroft - Walton can go up to 720 kV.

Maximum voltage attained with this type of cascade generator is about 4 MV.

Van de Graaff Accelerator :-



Principle of Van de Graaff generator:

Let us assume two spheres one inside the other each carrying charges q (inside) and Q (outside sphere).

The potential of the large sphere is caused in part by q and another part by its own charge Q. Thus

$$V_R = \frac{1}{4\pi\epsilon_0}\left(\frac{Q}{R} + \frac{q}{R}\right) \quad \text{--- (2)}$$

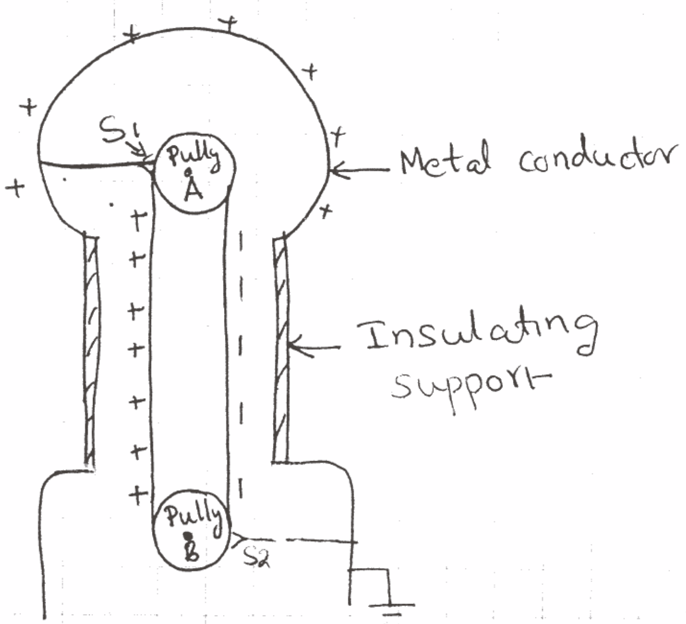
Similaraly the potential of the small sphere is

$$V_r = \frac{1}{4\pi\epsilon_0}\left(\frac{q}{r} + \frac{Q}{R}\right) \quad \text{—③}$$

The potential difference is

$$V = V_r - V_R = \frac{q}{4\pi\epsilon_0}\left(\frac{1}{r} - \frac{1}{R}\right) \quad \text{—④}$$

This clearly shows that the inner sphere is alwayas at higher potential with respect to the outer sphere. This implies that if we inject charges in to the inner sphere (somehow) it will always be made migrated to outer sphere because it is always at lower potential. irrespective of already existing charges on it. This is the principle of Van-de Graaff accelerators.



The pulleys A and B are made of different materials so that when the belt contacts with non-conducting pulleys the belt acquires +ve charges in one case and -ve charges in other case. Sharp points are connected at "S1" and "S2" to pick up charges.

The dielectric strength of air at atmospheric pressure air can not exceed 30kV|cm. If the sphere is about 1m in radius then the maximum voltage attainable is about 3 MV.

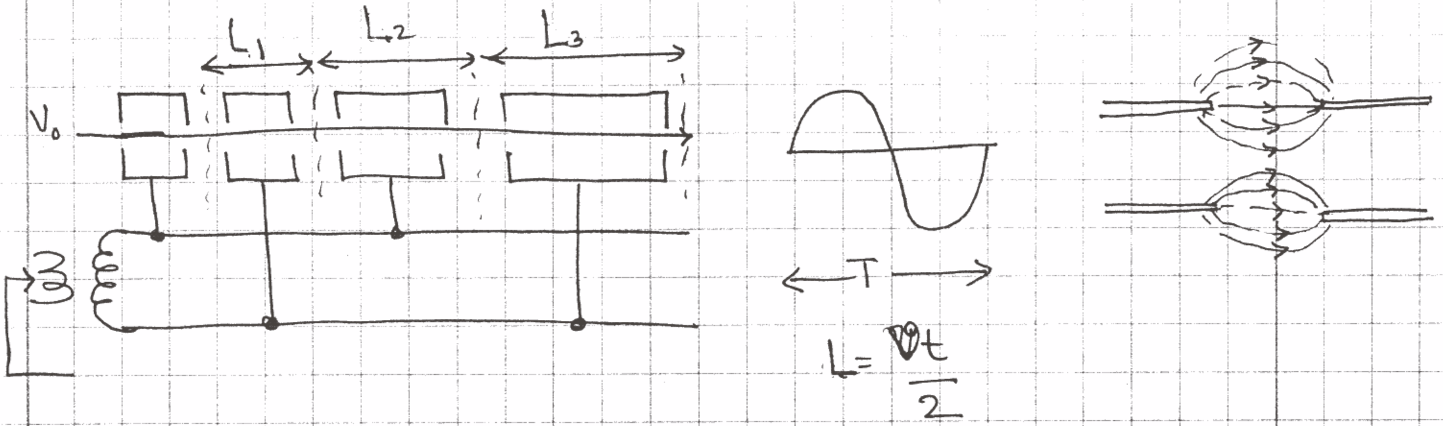
By imbedding the whole system in another container and pressuring using good insulating gas (Nitron or methane derivatives) one can go up or beyond 12MV.

Radiofrequency Linear Accelerators :-

The principle of radiofrequency linear accelerators is based on "alternating fields" and "drift tubes". These were proposed by Ising (1924) and Wideroe (1928). In these accelerators the particles will travel along nearly linear paths. Increment in particle energy occurs in an rf electric field in synchronous with the passage of particles in the gaps of a coaxial system of cylindrical electrodes or in the electric field of an electromagnetic standing or traveling wave induced in a wave guide.

Wideroe Accelerating Structure :-

Here charged particle beam move inside a system of cylindrical electrodes (called drift tube) of varied length. The drift tubes have an



an rf voltage applied to them so that the particles gets accelerated only at the gap and inside the cylindrical tubes they do not see any field. The successive electrode lengths are matched to the growing velocity of the particles.

The synchronism condition is

$$L_i \lesssim \frac{1}{2} v_i T_{rf} \quad ; \quad T_{rf} = \text{period of rf source} \quad \text{—— (5)}$$

v_i = velocity of the particle in i^{th} drift tube.

Reletivistically, for electrons the $v \to c$ and the distance between gaps becomes constant

$$L = \frac{1}{2} cT \quad \text{—— (6)}$$

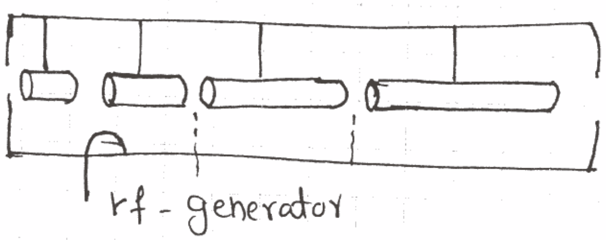
In Wideroe type of linear accelerators a potential difference of the order of thousand kilovolts can be attained in the gap. The radio frequency can operate at hundreds of or even thousands of mega-hertzs.

In 1931, Sloan and Lawrence built a Wideroe type of linear accelerator with

RF frequency = 10 MHz
Gap voltage = 42 kV
Drift tubes = 30

Accelerated mercury ions to a total Kinetic energy of 1.26 MeV.

At higher frequencies Wideroe structure becomes very lossy due to electromagnetic radiation. To overcome this problems Alvarez (1946) proposed to enclose Wideroe structure in metalic cavities.



Fermilab uses this type of linear accelerators, in the ealy part of Linac. Totally there are five tanks of this type. By the end of 5th tank the energy of H^- ions will be 116.5 MeV.

Modern accelerators make use of wave guides to establish electric field. A wave guide is a pipe of conducting material in which an oscillating electromagnetic field is established. Electromagnetic field can form a standing wave in the cavity. Standing wave is superposition of two travelling wave moving in opposit direction. An ion that moves with the same velocity as the traveling wave is subjected to a constant accelerating force.

In a circular metalic tube the phase velocity V_{ph} is always greater than velocity of light i.e.,

$$V_{ph} > c \quad \text{—(7)}$$

The group velocity V_g, is always

$$V_g < c \quad \text{—(8)}$$

To accelerate charged particles in wave guide we must have

$$V_{ph} = V_{particle}. \quad \text{—(9)}$$

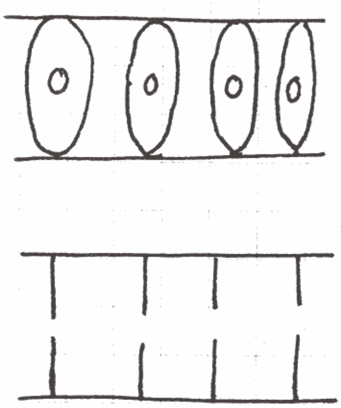
Thus for efficient acceleration in a waveguide we must modify wave guide structure.

This can be done by inserting metallic structures into aperture of the circular wave guide - "disk loaded waveguide"

This is a common structure for electron linear accelerators.

By proper selection of geometric dimensions and disks the frequency can be choosen and $V_{ph} \cong V_p$ can be achieved.

<u>Energy Gain</u>:-



Let the particle be at the center of accelerating gap at time $t=0$ and its velocity is v. Since the cavity fields oscillate, the potential energy gain is

$$eV(t) = e\hat{V}_0 \cos \omega t = e\hat{V} \cos\left(\omega \frac{s}{v}\right)$$

where $s = vt$; assuming $\Delta v \cong 0$

Then energy gain in a cavity of length l_c is

$$\Delta E_{kin} = \int_{-\frac{l_c}{2}}^{l_c/2} \frac{e\hat{V}_0}{l_c} \cos\left(\omega \frac{s}{v}\right) . ds$$

$$\Delta E_{kin} = e\hat{V}_0 \frac{\sin \frac{\omega \lambda_{rf}}{4v}}{\frac{\omega \lambda_{rf}}{4v}} = e\hat{V}_0 T_t \quad \text{—(10)}$$

where $l_0 = \lambda_{rf}/2$

T_t is called "transit time factor". For $v \ll c$, T_t and ΔE_{kin} is small and hence accelerating cavity structures of above type are not advantageous. While for $v \simeq c$, the wave guide structure is very advantageous.

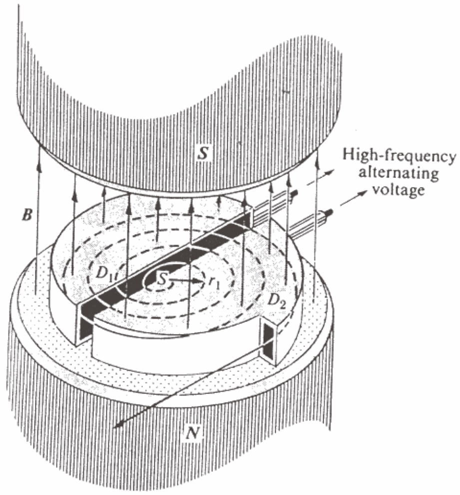
At Fermilab, last 7 tanks are "disk loaded side coupled" wave guides, which accelerate H^- from 116.5 MeV to 400 MeV and have frequency of 800 MHz.

RFQ: Radio Frequency Quadrupoles.

This is an ion accelerator in which both acceleration and transverse focusing are performed by rf fields.

Cyclic Accelerators

Cyclotron :- The cyclotron is developed by E.O. Lawrence and M.S. Livingston in 1931. at Univ. of Berkeley.



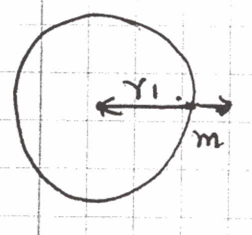
The heart of the cyclotron is a pair of metal chambers called "Dees" inserted between pole pieces of a dipole magnet. as shown in the figure. At the mid point of the gap an ion source will be located. The dees are connected to the terminals of a circuit generating alternating voltage. particles are accelerated between dee-gaps and within the dees they are bent by the applied magnetic field.

principle of cyclotron:

consider an ion of charge q and mass m. The ion is accelerated by the electric field in the gap between dees and acquire an initial velocity v_1. Charged particle with velocity $\perp^{er}$ to the magnetic field B will experience Lorentz force $\frac{q v_1 B}{c}$ which is balanced by centrifugal force $\frac{m v_1^2}{r_1}$

Thus

$$\frac{m v_1^2}{r_1} = q \frac{v_1 B}{c} \quad \text{— (11)}$$



or angular velocity ω is given by

$$\omega = \frac{v_1}{r_1} = \left(\frac{q}{mc}\right) B \quad \text{— (12)}$$

Hence angular velocity is independent of the speed of the ion and radius of the circle. Therefore, if electric field reverses at regular intervals, each equal to the time required for the ion to make a half revolution, the field in the gap will always be in proper direction to accelerate an ion each time the gap is crossed. This unique feature makes cyclotron to work.

Thus, if R is radius of dees then

$$v_{max} = \frac{BR}{c}\frac{q}{m} \; ; \; p = \frac{qBR}{c} \quad \text{— (13)}$$

↑ This is relativistically correct

and corresponding kinetic energy of the ion is

$$E_{kin} = \frac{1}{2} m \left(\frac{q}{mc}\right)^2 B^2 R^2 \quad \text{— (14)}$$

$$= \frac{q^2 B^2 R^2}{2 mc^2}$$

Synchrocyclotron:- Veksler (Soviet Union) McMillan (USA) (1945)

There are two reasons why conventional cyclotrons run into difficulties at high energies

1. ω is independent of particle speed only if $v \ll c$ - velocity of light.
As speed increases we must use relativistic mass.

2. $E_{kin}(max) \sim R^2$ i.e size of the magnet becomes exceedingly large. e.g., $B = 1.5T$
$R = 65m$ for a 30 GeV proton.

The relativistic modification will be

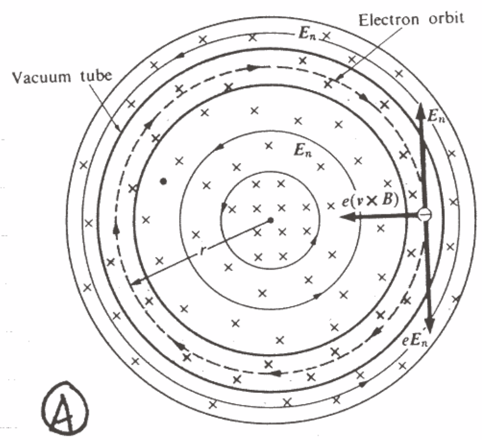
$$\omega = B\left(\frac{q}{m}\right)\sqrt{1 - v^2/c^2} \quad \text{--- (15)}$$

Thus the resonance will be ensured by synchronously decreasing ω during acceleration such a way that $\frac{\omega . m}{\sqrt{1-v^2/c^2}}$ remains constant.
Accelerators which use this technique is called as synchrocyclotron. or frequency modulated cyclotrons or Phasotron (Russian name)

Largest synchrotron built upto date accelerates protons to 1 GeV (Leningrad Institue of Nuclear physics)

Betatron :- (Donald W. Kerst, IL 1941)

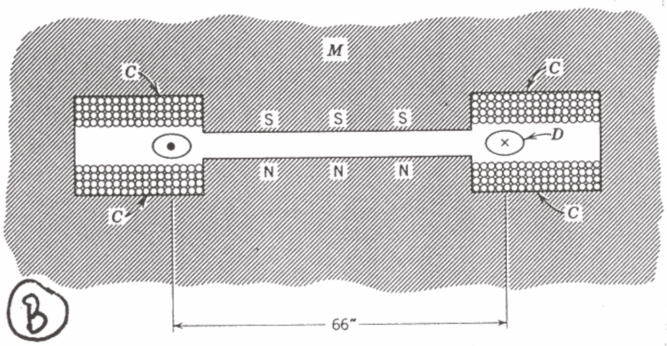
According to Maxwell's electromagnetic equations "changing magnetic field induces an electric field" (Faraday's law of electricity) This is the principle of a betatron. Thus,



(A)

Fig. 33–12 An electron in the vacuum tube of a betatron is accelerated by the induced electric field E_n and forced to move in a circle by the magnetic field B.

$$\oint_L E \cdot dl = -\frac{d\phi_B}{dt} \quad (26c)$$



(B)

The accelerators which are based on this principle are also called as "Magnetic Induction Accelerators". The figures (A) and (B) illustrate top and cross-sectional views of a betatron.

Betatron consists of a "doughnut" shaped accelerating chamber located between the pole pieces of an electromagnet. Electrons emitted by a source move in a circular path. Unlike in a cyclotron there are no accelerating electrodes. The changing magnetic field in the electromagnet performs two functions viz.,

1. It causes the electrons to bend and keeps them in a circular orbit

2. It induces an accelerating voltage because it has a time varying flux.

To keep a particle in its orbit in the doughnut we have magnetic field B_o such that

$$P = \frac{e}{c} B_o R \quad \text{— (16)}$$

— R is radius of the orbit.

P = momentum.

This equation is relativistically correct.

The magnetic field flux density is increasing i.e., changing with respect to time. Then from Eq. 26c

$$\oint E\,ds = 2\pi R E = -\frac{1}{c}\frac{d\phi}{dt} \quad \text{— (17)}$$

Hence the momentum increases according to

$$eE = \frac{dp}{dt} = -\frac{e}{2\pi R c}\frac{d\phi}{dt} \quad \text{— (18)}$$

Let $\langle B \rangle$ be the average value of magnetic field over the area enclosed by the orbit then

$$\phi = \pi R^2 \langle B \rangle \quad \text{— (19)}$$

At constant radius

$$\frac{d\phi}{dt} = \pi R^2 \frac{d\langle B \rangle}{dt} \quad \text{— (20)}$$

Then with eq^n (18)

$$\frac{dp}{dt} = -\frac{eR}{c2}\frac{d\langle B \rangle}{dt} \quad \text{— (21)}$$

on the other hand with eqⁿ (16) with constant R

$$\frac{dp}{dt} = -\frac{e}{c} R \frac{dB_0}{dt} \qquad \text{---} \quad (22)$$

Thus (22) and (21) give

$$2\frac{dB_0}{dt} = \frac{d\langle B \rangle}{dt} \qquad \text{---} \quad (23)$$

This is the fundamental condition for the betatron.

The final momentum is given by

$$p = p_0 - \frac{e}{2\pi R c}(\phi - \phi_0) \qquad \text{---} \quad (24)$$

Note that e is electronic charge of an electron becacuse we started with an electron.

synchrotrons :-

In recent years high energy accelerators have been constructed in which magnetic field as well as rf frequency vary synchronously. Such accelerators are called as synchrotrons. Fermilab has five such synchrotrons in the complex and one fixed frequency synchrotron. They are Booster (8 GeV), Main Injector (150 GeV), Tevatron (1 TeV), Debuncher (8 GeV) Accumulator ring (8 GeV) and fixed frequency synchrotron in Recycler Ring (8 GeV).

Table 3.1. Parameter disposition for different acceleration principles

principle	energy γ	velocity v	orbit r	field B	frequency f_{rf}	flux
Cyclotron:	1	var.	$\sim v$	const.	const.	cont.[a]
Synchro cyclotron:	var.	var.	$\sim p$	$B(r)$	$\sim \frac{B(r)}{\gamma(t)}$	pulsed
Isochron cyclotron:	var.	var.	$r = f(p)$	$B(r, \varphi)$	const.	cont.[a]
Proton/Ion-synchrotron:	var.	var.	R	$\sim p(t)$	$\sim v(t)$	pulsed
Electron-synchrotron:	var.	const.	R	$\sim p(t)$	const.	pulsed

[a] continuous beam, but rf modulated