Photoelectric Effect

- First explained by Einstein in 1905.
- An interaction between photon and the atom

$$\gamma + \text{atom} \rightarrow \text{atom}^+ + e^-$$

- Photons with energy $E_\gamma >> E_b$, the binding energy of an electron in the atom, may be absorbed and an atomic electron ejected with kinetic energy $T = E_\gamma - E_b$.
- "Resonant" absorption occurs with E_γ near the bound state energies of the electrons,

$$E_n = -13.6\,eV \cdot \frac{Z^2}{n^2} \quad \left(\begin{array}{l} n = \text{principal} \\ \text{quantum number} \end{array}\right).$$

- The photoelectric cross section

$$\sigma_{pe} = \frac{32\pi}{3} \cdot \sqrt{2} \cdot Z^5 \alpha^4 \left(\frac{m}{\hbar\omega}\right)^{7/2} \cdot (\alpha \lambda\!\!\!^-)^2$$

$$= \left(\frac{32}{\epsilon^7}\right)^{1/2} \cdot \alpha^4 Z^5 \cdot \sigma_{Th}$$

$$\sigma_{Th} = \frac{8}{3}\pi r_e^2$$

$$\sigma_{pe} \propto \frac{Z^5}{E_\gamma} \quad \text{for high energies}$$

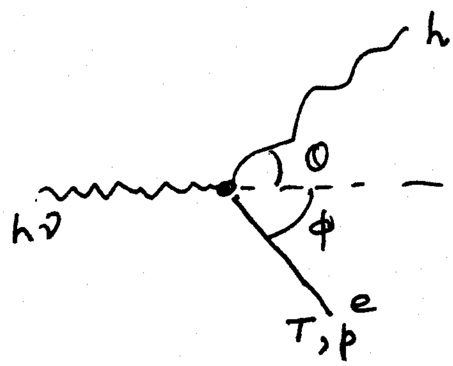
$$\alpha \lambda\!\!\!^- = \frac{e^2}{\hbar c} \cdot \frac{\hbar}{m_e c} = \frac{e^2}{m_e c^2} = r_e$$

- Main application in PMTs

Compton Effect

- Scattering of an incident photon off a quasi-free atomic electron.

$$\gamma + e \rightarrow \gamma' + e'$$



From energy momentum conservation,

$$h\nu = h\nu' + T$$

$$\frac{h\nu}{c} = \frac{h\nu'}{c}\cos\theta + p\cos\phi$$

$$0 = \frac{h\nu'}{c}\sin\theta - p\sin\phi$$

Using these and

$$T = \sqrt{p^2c^2 + m_e^2c^4} - m_ec^2,$$

One can derive a number of useful relations.

The frequency of the scattered photon:

$$\nu' = \frac{\nu}{1+\epsilon(1-\cos\theta)} \quad \text{—— (1)} \qquad \epsilon = \frac{h\nu}{m_e c^2}$$

The kinetic energy of the recoil electron:

$$T = m_e c^2 \cdot \frac{\epsilon^2(1-\cos\theta)}{1+\epsilon(1-\cos\theta)} \quad \text{—— (2)}$$

The electron recoil angle:

$$\cos\phi = (1+\epsilon)\left[\frac{1-\cos\theta}{2+\epsilon(\epsilon+2)(1-\cos\theta)}\right]^{1/2} \quad \text{—— (3)}$$

For back scattering $(\theta = \pi)$, (2) becomes,

$$T = T^{max} = \frac{2\epsilon^2}{1+2\epsilon} \cdot m_e c^2$$

For $\epsilon >> 1$, i.e., $E_\gamma = h\nu >> m_e c^2$,

$$T^{max} \approx E_\gamma$$

The cross section for compton scattering can be calculated using QED.

Klein - Nishina formula:

$$\frac{d\sigma}{d\Omega} = \frac{r_e^2}{2} \cdot \frac{1}{[1+\epsilon(1-\cos\theta)]^2}\left(1+\cos^2\theta + \frac{\epsilon^2(1-\cos\theta)^2}{1+\epsilon(1-\cos\theta)}\right)$$

$$\sigma_c = \int \frac{d\sigma}{d\Omega} \cdot d\Omega$$

(Probability/electron for a compton scattering to occur.)

$$\sigma_{cs} = \frac{E_\gamma'}{E_\gamma} \cdot \sigma_c$$

scattered cross section

$$\sigma_{ca} = \sigma_c - \sigma_{cs}$$

absorption cross section

Pair Production

Production of electron-positron pairs is possible in the coulomb field of a nucleus (or an atomic electron) if $E_\gamma > E_{th}$

From energy & momentum conservation, the threshold energy can be calculated.

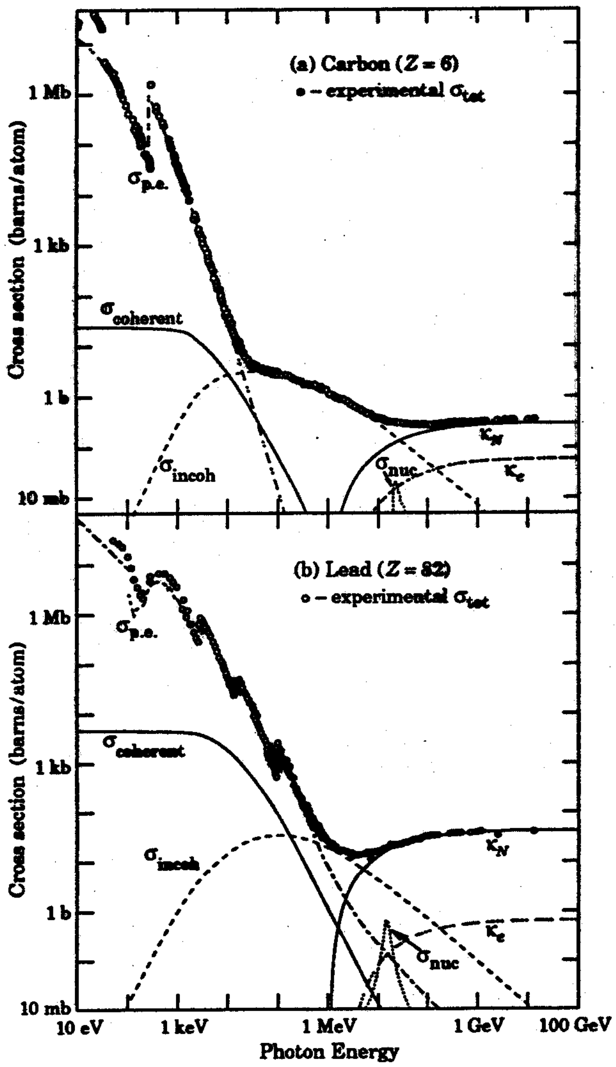
$$E_\gamma \geq 2 m_e c^2 + 2 \cdot \frac{m_e^2}{m_{nucleus}} c^2$$

But $m_{nucleus} >> m_e$

$\therefore E_\gamma \geq 2 m_e c^2$ is the threshold

$$\sigma_{pair} = 4 \alpha r_e^2 Z^2 \left(\frac{7}{9} \ln \frac{183}{Z^{1/3}} \right) \text{ cm}^2/\text{atom}$$

$$\boxed{\sigma_{pair} \approx \frac{7}{9} \cdot \frac{A}{N_A} \cdot \frac{1}{X_0}}$$



$$I = I_0 \, e^{-\mu x}$$

$$\frac{\mu}{\rho} = \frac{N_A}{A} \cdot \sum_i \sigma_i = \frac{N_A}{A} \cdot \left(\sigma_{PE} + Z\,\sigma_{comp} + \sigma_{pair} \right)$$

Mass attenuation coefficient

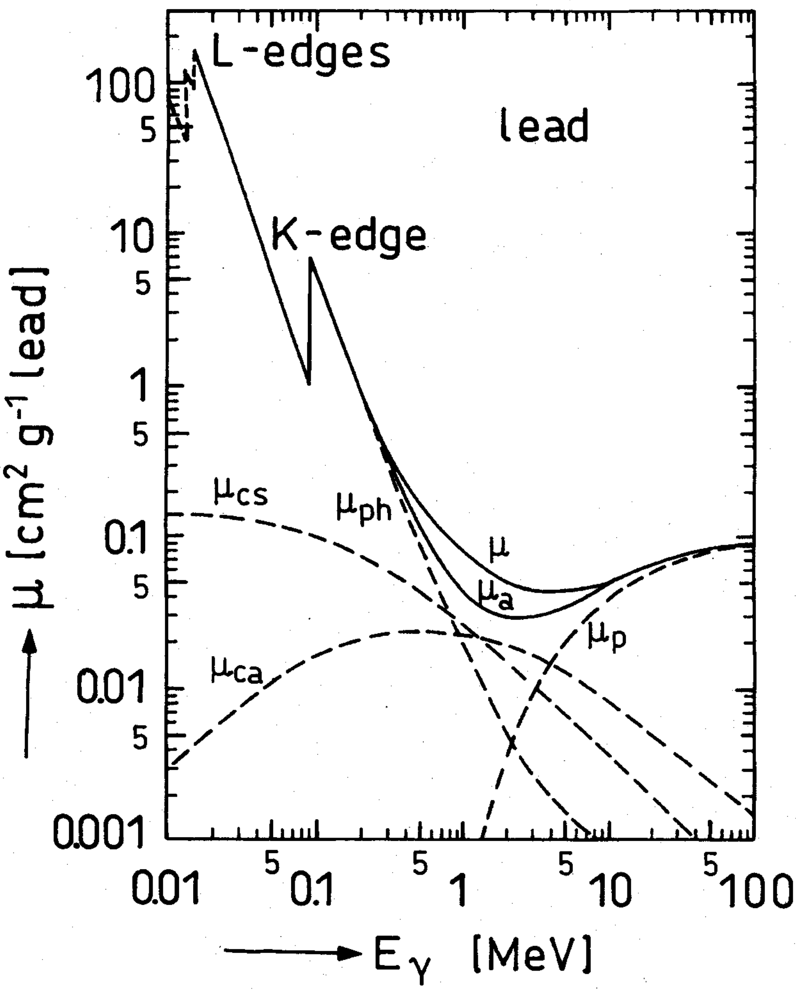


Fig. 1.14 d. Energy dependence of the mass attenuation coefficient μ and mass absorption coefficient μ_a for photons in lead [63, 73, 74, 75].

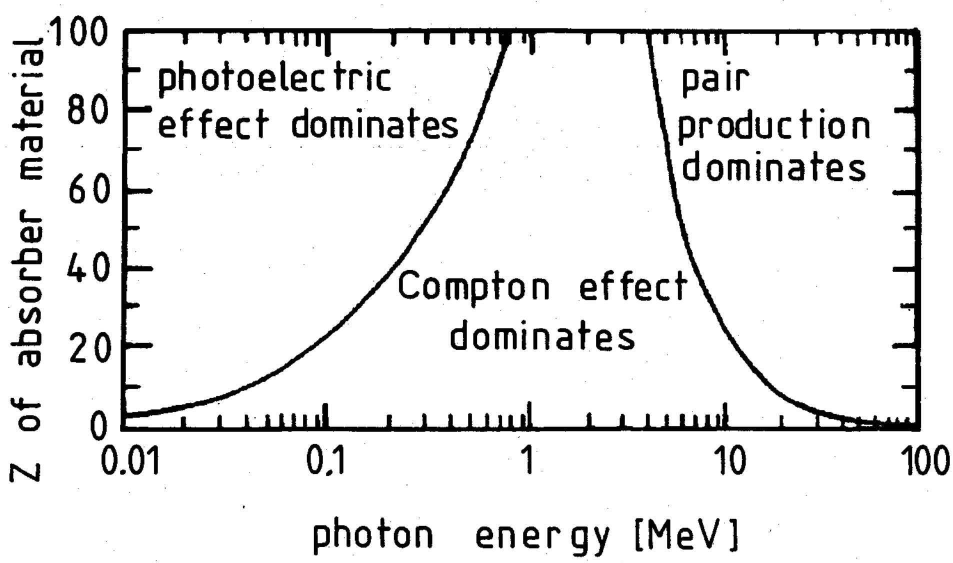


Fig. 1.15. Ranges, in which the photoelectric effect, Compton effect, and pair production dominate as a function of the photon energy and the target charge Z [37, 65, 68].

Strong Interactions

Responsible for particle creation reactions, hadronic cascade development.

$$\sigma_{inel} \simeq \sigma_0 A^{\alpha} \qquad \alpha = 0.71$$

Nuclear Absorption or Interaction length

$$\lambda = \frac{A}{N_A \cdot \rho \cdot \sigma_{inel}} \quad (cm)$$

$\lambda = 39$ cm Al
10 cm U
25 cm Fe

Charged Particle Multiplicity

$$\langle N \rangle \sim a \ln \sqrt{s} + b$$