Octupole Name	Number
TOZD	24
TOZF	12
TOF39S	7
TOD39S	8
TOF39C	8
TOD39C	8

Table 1: Octup[oles in the Tevatron. (TOF39S, TOD39S) are on the same bus but the TOD39S are rotated by 45^o, thus reversing its polarity. Similay (TOF39C,TOD39C).

$$
\begin{aligned}
\Delta\nu_x &= a_1 J_x + a_2 J_y \\
\Delta\nu_y &= a_2 J_x + a_3 J_y \\
a_1 &= \frac{1}{16\pi}\sum_n k_3(n)\beta_{x,n}^2 \\
a_2 &= -\frac{1}{8\pi}\sum_n k_3(n)\beta_{x,n}\beta_{y,n} \\
a_3 &= \frac{1}{16\pi}\sum_n k_3(n)\beta_{y,n}^2
\end{aligned}
$$

Coefficience	TOZF ,TOZD polaority			
	(+) (+)	(+) (-)	(-)(+)	(-) (-)
$\frac{\partial\nu_x}{\partial(a_x^2)}$	>0	>0 or <0	<0 or >0	<0
$\frac{\partial\nu_y}{\partial(a_y^2)}$	>0	$\leq$ 0or >0	>0 0r <0	<0
$\frac{\partial\nu_x}{\partial(a_y^2)}$	<0	<0 or >0	<0 or >0	>0
$\frac{\partial\nu_y}{\partial(a_x^2)}$	<0	<0 or >0	<0 or >0	>0

Table 2: possible contribution to the incoherent tune spread by octupoles

Let $k_3(F)$ be TOZF's strength, $k_3(D)$ be TOZD's strength, and the average β-functions is 93.6 m in x-plane, 30.2 m in y-plane for TOZF, 30.5 m in x-plane, 92.5 m in y-plane for TOZD, we can find the action-dependent tune-shift due to octupoles are as follows:

$$\begin{aligned}\Delta\nu_x &= 2091.5251\left\{[k_3(F)+0.2124k_3(D)]J_x - 0.6444[k_3(F)+1.9961k_3(D)]J_y\right\}\\ \Delta\nu_y &= 1349.6594\left\{-[k_3(F)+1.9961k_3(D)]J_x + 3.0269[0.0533k_3(F)+k_3(D)]J_y\right\}\end{aligned}$$

$J_{x,y}$ is the action,and related to the amplitude by

$$J_{x,y} = \frac{(a_{x,y})^2}{2}\cdot\epsilon_{nx,y}$$

here $a_{x,y}$ is the amplitidue im units of beam size σ, $\epsilon_{nx,y}$ is the normalized emittance (1σ)in x or y plane.

If we consider the particles along the diagonal, then $J_x = J_y$. At present machine conditions, the proton emittance is 25πmm-mrad(95%), and if we look at the the particle with initial ampplidute of 5σ, then

$$
\begin{aligned}
J_{x,y} &= \frac{25^2}{2} \cdot \frac{25\pi \times 10^{-6}}{6\pi} \frac{1}{159.8683} \\
&= 0.3258 \times 10{-}6
\end{aligned}
$$

Then,we can get the expression for the octupole's strength needed to compensate the decrease due to lowring the chromaticities: The octupole strengths now can be calculated as follows:

$$
\begin{aligned}
k_3(F) &= -\frac{1.4897 \times 10^{-3}}{J_{x,y}}[0.6194 \times \Delta\nu_x + \Delta\nu_y] \\
&= -4.5726 \times 10^3[0.6194 \times \Delta\nu_x + \Delta\nu_y] \\
k_3(D) &= -\frac{0.7508 \times 10^{-3}}{J_{x,y}}[\Delta\nu_x + 0.6570 \times \Delta\nu_y] \\
&= -2.3044 \times 10^3[\Delta\nu_x + 0.6570 \times \Delta\nu_y]
\end{aligned}
$$

The coherent stability condition [bill.Ig lecture]

$$\Delta\nu_s = \frac{\Delta\Omega_s}{\omega_o}$$
$$\Delta\Omega_s > \Lambda[l_s]$$

$\Lambda[l_s]$ **is the growth rate of the head-tail modes.**

The growth rate $\Lambda^{[l_o]}(\chi = -1.5)$ **at injetion energy E=150 Gev is about 120**$sec^{(-1)}$ **by measurement, 55** $sec^{(-1)}$ **by calculation from model. Here** $\chi = \xi \cdot l_s\omega_o/\eta$

the coherent tune shift is

$$\Delta\nu_{coh}^{l=0} = \frac{\Delta\Omega_0}{\omega_o} \approx 4 \times 10^{-4}(?)$$

On the other hand, the synchrotron tune spread at injetion energy E=150 Gev is calculated as follows:

$$\Delta\nu_s \approx \nu_{s0} \cdot \frac{1}{16}\left(\langle\phi\rangle\right)^2 \approx 2.2 \times 10^{-4}$$

$\langle\phi\rangle \approx 1.4rad, \nu_{s0} = 1.8 \times 10^{-3}$ **at injection. The incoherent linear tune shift due to the space charge for the particle near the center of the proton bunch with 3-D Gaussian density distribution is**

$$(\Delta\nu)_{spach} = -1.25 \times 10^{-3}$$

Therefore, in order for the landau damping to work, the betatron tune spread should be

$$\begin{aligned}(\Delta\nu)_\beta &= [(\Delta\nu)_{spech} + \Delta\nu_{coh}] - \Delta\nu_s \\ &= 1.25e - 3\end{aligned}$$

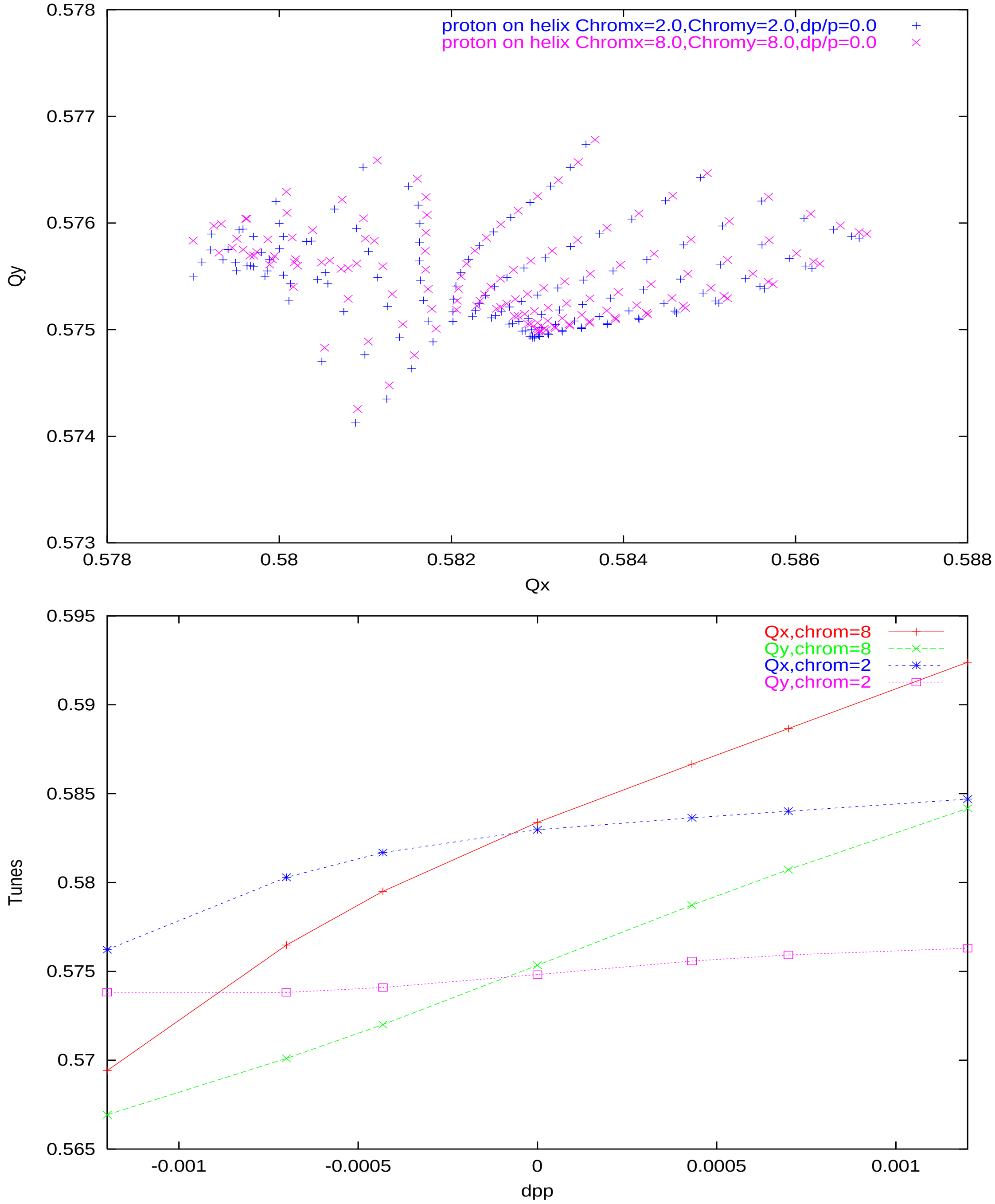


Figure 1: Tune spread due to nolinearities and chromaticities

Chromaticities	Qx'=2,Qy'=2		Qx'=8,Qy'=8		Qx'=1.7,Qy'=2.1,T:SF=2,T:SD=1	
$\nu_{max} - \nu_{min}$	x-plane	y-plane	x-plane	y-plane	x-plane	y-plane
due to nonlinearities	0.0076336	0.0022995	0.0078291	0.0022114	0.0041682	0.0048774
due to chromaticities,dpp=±0.431e-3	0.001951	0.001491	0.007159	0.006724	0.003245	0.001287
due to chromaticities,dpp=±0.7e-3	0.003720	0.002107	0.012185	0.010619	0.005562	0.003067
due to chromaticities,dpp=±1.2e-3	0.008474	0.002478	0.022979	0.017241	0.008973	0.004987

Table 3: Dynamic aperture vs. chromaticities without/with octupoles

Without octupoles

$$\Delta(\Delta\nu_x)_{fp} = -0.0001955 \quad \Delta(\Delta\nu_y)_{fp} = 0.000088$$
$$\Delta(\Delta\nu_x)_{p=0.431e-3} = -0.005208 \quad \Delta(\Delta\nu_y)_{p=0.7e-3} = -0.005233$$
$$\Delta(\Delta\nu_x)_{p=0.7e-3} = -0.008465 \quad \Delta(\Delta\nu_y)_{p=0.7e-3} = -0.008512$$
$$\Delta(\Delta\nu_x)_{p=1.2e-3} = -0.014505 \quad \Delta(\Delta\nu_y)_{p=1.2e-3} = -0.014763$$

With octupoles

$$\Delta(\Delta\nu_x)_{fp} = -0.0036609 \quad \Delta(\Delta\nu_y)_{fp} = 0.002666$$
$$\Delta(\Delta\nu_x)_{p=0.431e-3} = -0.003914 \quad \Delta(\Delta\nu_y)_{p=0.4e-3} = -0.005437$$
$$\Delta(\Delta\nu_x)_{p=0.7e-3} = -0.006623 \quad \Delta(\Delta\nu_y)_{p=0.7e-3} = -0.007552$$
$$\Delta(\Delta\nu_x)_{p=1.2e-3} = -0.014006 \quad \Delta(\Delta\nu_y)_{p=1.2e-3} = -0.012254$$

For the protons with the emittance of $25\pi mm - mrad(95\% normalized)$, we track the particles with the amplitude of $x = 5\sigma_x$ $y = 5\sigma_y$ and the momentun deviation dp/p=$1 * \sigma_p$ in two cases: with the chromaticities of 8 units and with the chromaticities of 2 units. Then, we get the differece of the tune shifts. This difference is supposed to be compensated by the octupoles

Figure 1. gives the footprint for the particle up to the amplitude of 5σ, we can see that the footprint was smaller in the case of the chromticities of 2 units than in the case of the chromticities of 8 units. The estimated value for the tune spread is about 0.00148 in x-plane, -0.001152 in y-plane. Then, the calculated octupole strengths for TOZF and TOZD are

$$\begin{aligned} k3(F) &= 7.4 \\ k3(D) &= 3.8 \end{aligned}$$

At tevatron injection,

$$\begin{aligned} \frac{1}{6}(B_3L/I) &= 102.3T/m^2/Amps \\ k3 = K_3L &= (B_3L/I)\cdot\frac{1}{(B\rho)}I \\ &= \frac{(6\times 102.3T/m^2/Amps)}{(3335.64T\cdot m/Tev \times 0.15Tev)}I \\ &= 1.2267506/m^3/Amps \times I \end{aligned}$$

I is the magnet currents of octupoles.

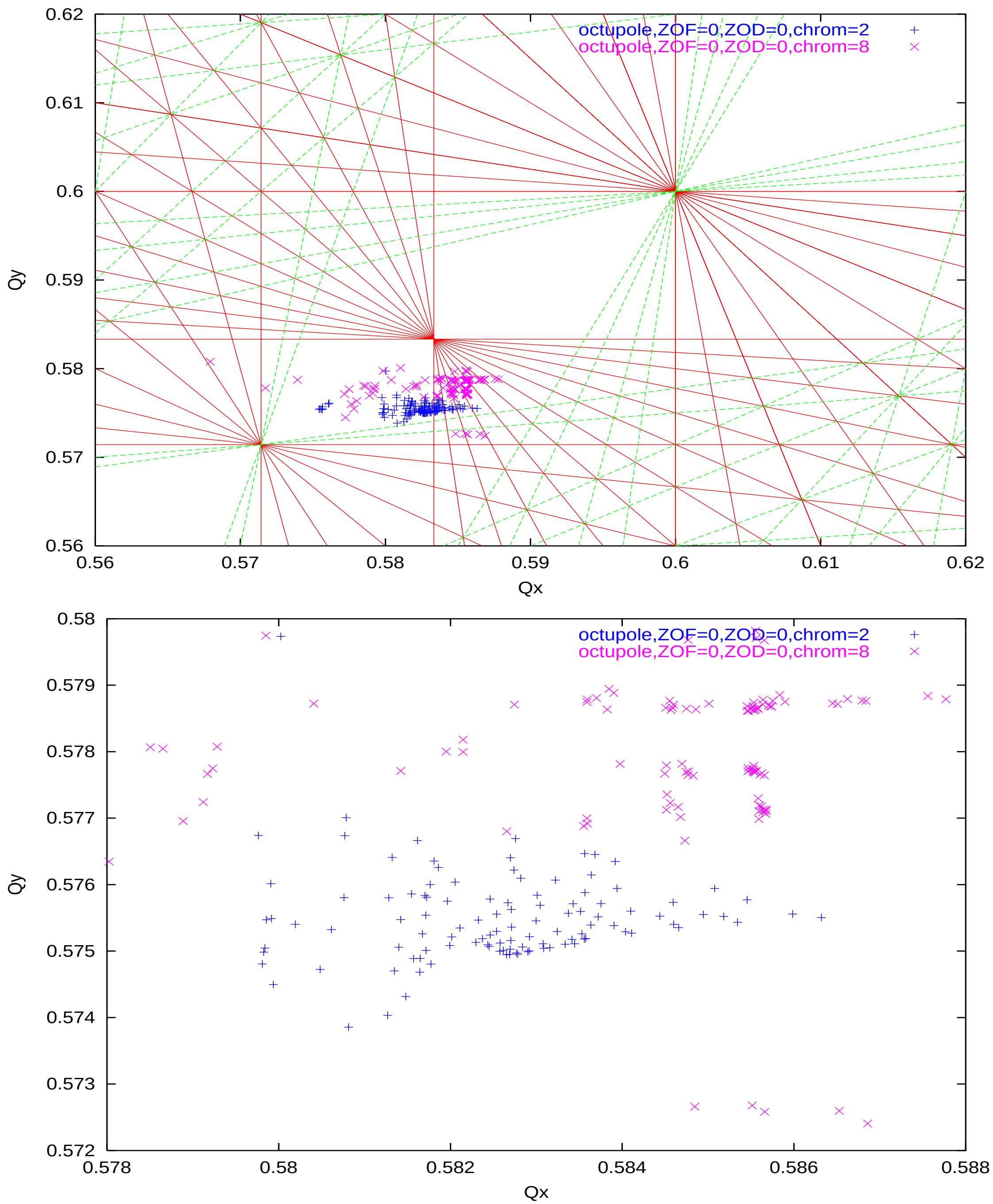


Figure 2: the footprint for the particle up to the amplitude of 5*sigma

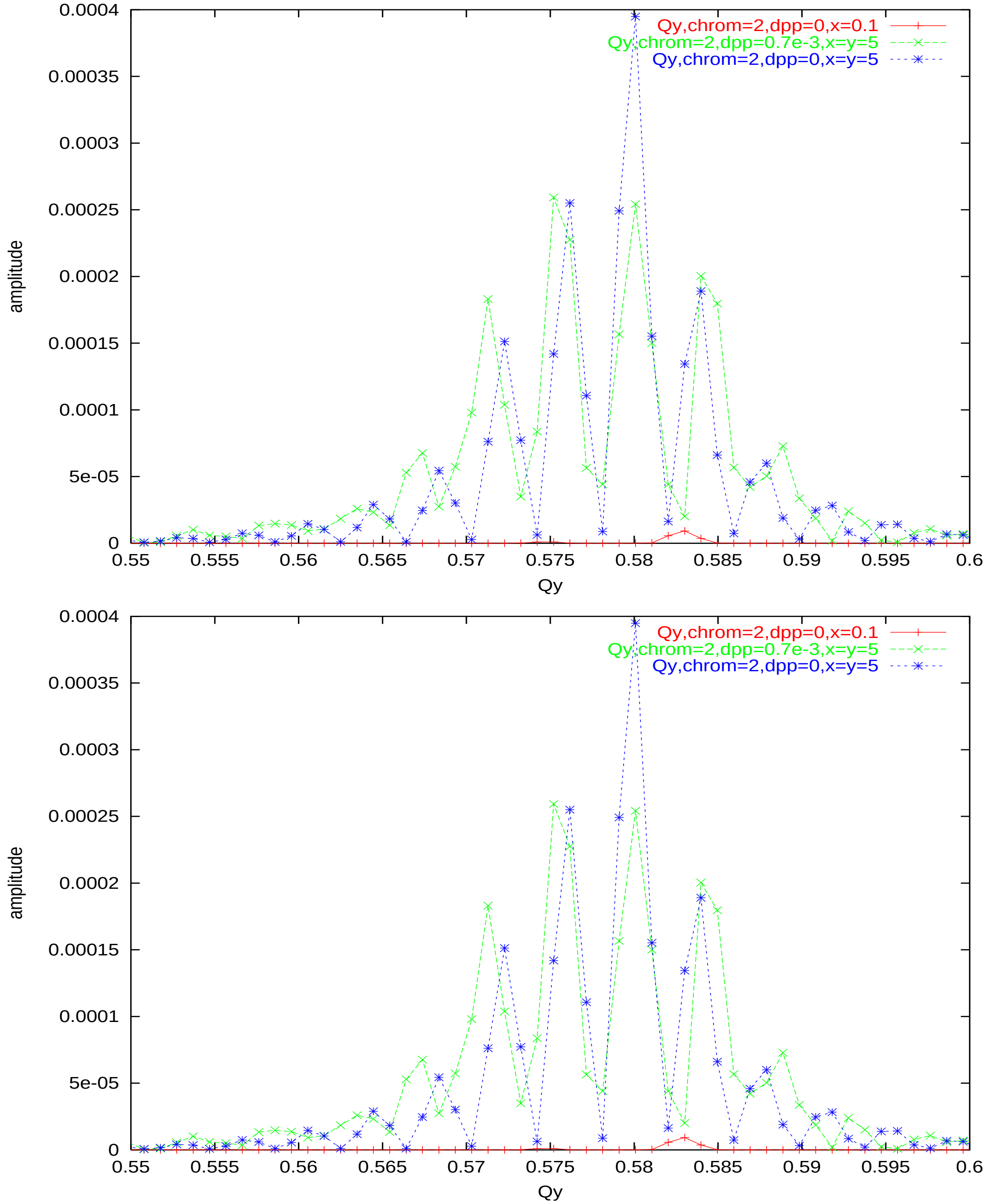
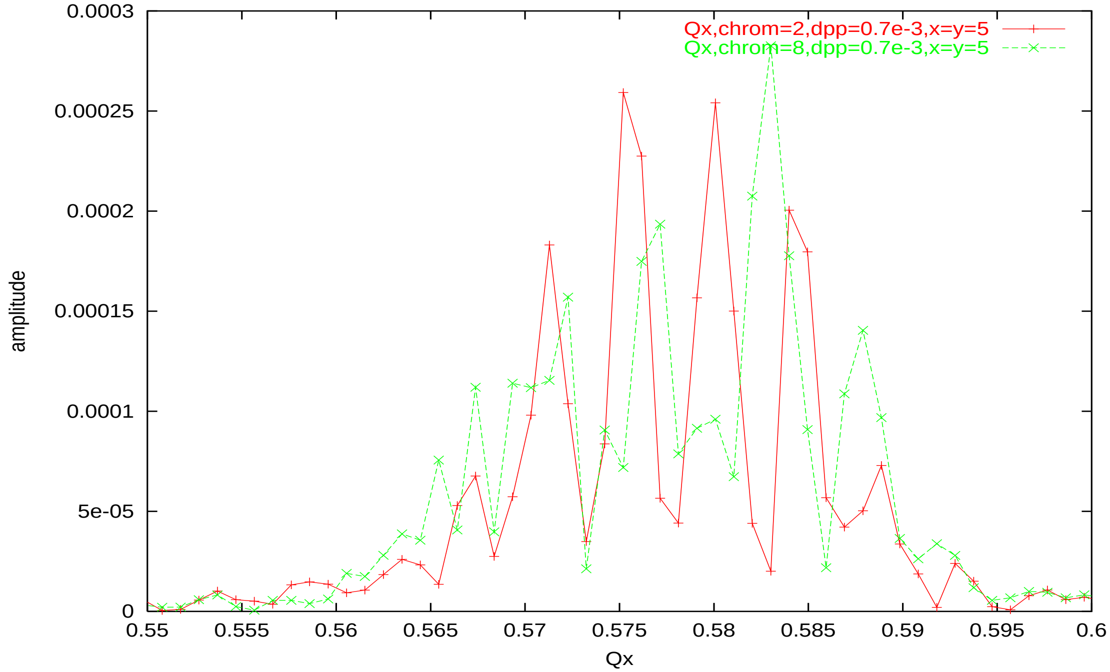


Figure 3: the footprint for the particle up to the amplitude of 5*sigma



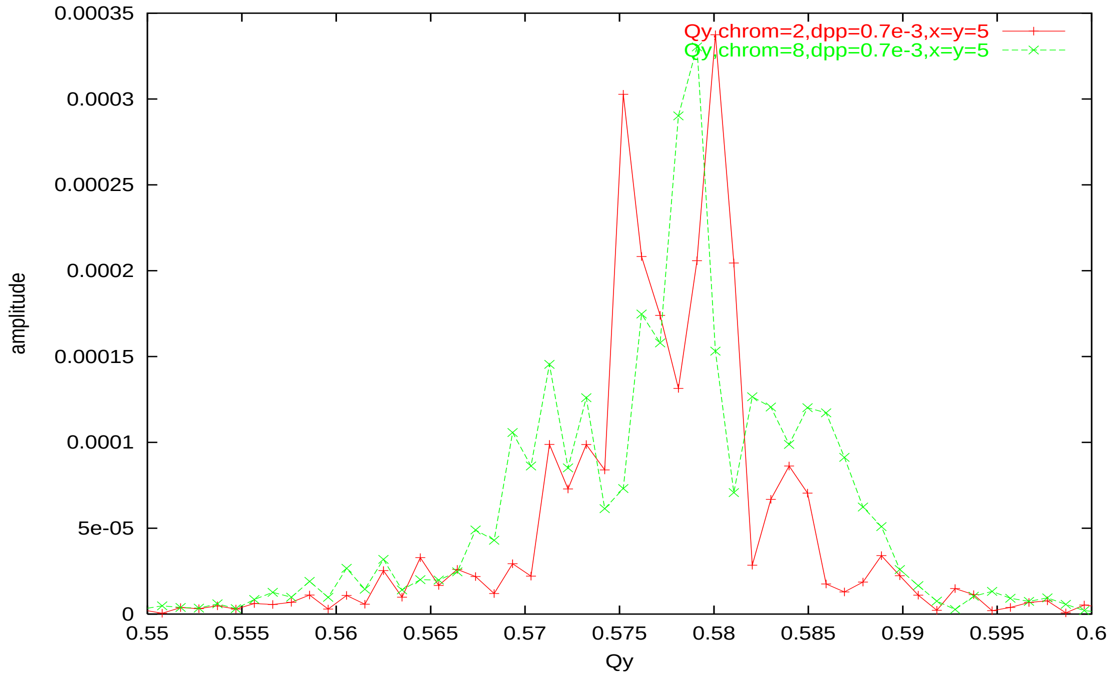


Figure 4: the footprint for the particle up to the amplitude of 5*sigma

Chromaticities	Qx'=8,Qy'=8	Qx'=2,Qy'=8	Qx'=2,Qy'=2	Qx'=0,Qy'=0
Tunes	(.58307,.5749)	(0.5829,0.5749)	(0.5831,0.5745)	(0.5831,0.5744)
T:SF	-0.02485	-0.02343	-0.02308	-0.02248
T:SD	0.00166	0.001083	-0.00130	-0.00229
<Chaotic Boder I>	4.7	4.7	5.2	4.7
Min.Chaotic Boder I	4.1	4.1	4.4	4.4
<Chaotic Boder II>	4.8	5.0	5.2	5.4
Min.Chaotic Boder II	4.1	4.5	4.4	4.6
<Lost I>	6.5	6.9	6.7	6.8
Min. Lost I	6.0	6.1	6.3	5.7
<Lost II>	6.4	6.9	6.8	6.9
Min. Lost II	6.0	6.5	6.3	6.2

Table 4: Dynamic aperture vs. chromaticities without octupoles

With the following strength

$$T:SF = -0.024587,$$
$$T:SD = 0.002085$$

$$\begin{array}{rl} MAD & SIXTRAC: \\ Qx' = 8.0, & Qx' = 6.4 \\ Qy' = 8.0 & Qy' = 9.5 \end{array}$$

Page T55 in the machine:

$$T:SF := 0.2593 * \Delta C_H,$$
$$T:SD := -0.1366 * \Delta C_v$$

Coefficience	TOZF ,TOZD polaority				no octupoles	
	(+2) (+1)	(+2) (-1)	(-2)(+1)	(-2) (-1)	Qx'=2,Qy'=2	Qx'=8,Qy'=8
$\frac{\partial\nu_x}{\partial(a_x^2)}$	-0.0001538	-0.0000815	-0.0000429	0.0001060	-0.0002321	-0.0000429
$\frac{\partial\nu_y}{\partial(a_y^2)}$	0.00006336	-0.0000504	-0.0000405	-0.0001491	0.0000282	-0.0000529
$\frac{\partial\nu_x}{\partial(a_y^2)}$ or $\frac{\partial\nu_y}{\partial(a_x^2)}$	0.0000500	-0.00000034	-0.0000529	-0.0001236	0.0000669	-0.0000405
Average DA	6.98	7.14	6.6	6.24	6.8	6.4
minimum DA	6.2	6.3	5.9	5.4	6.3	6.0
Chaotic border	5.86	5.48	4.46	4.42	5.2	4.7

Table 5: possible contribution to the incoherent tune spread by octupoles

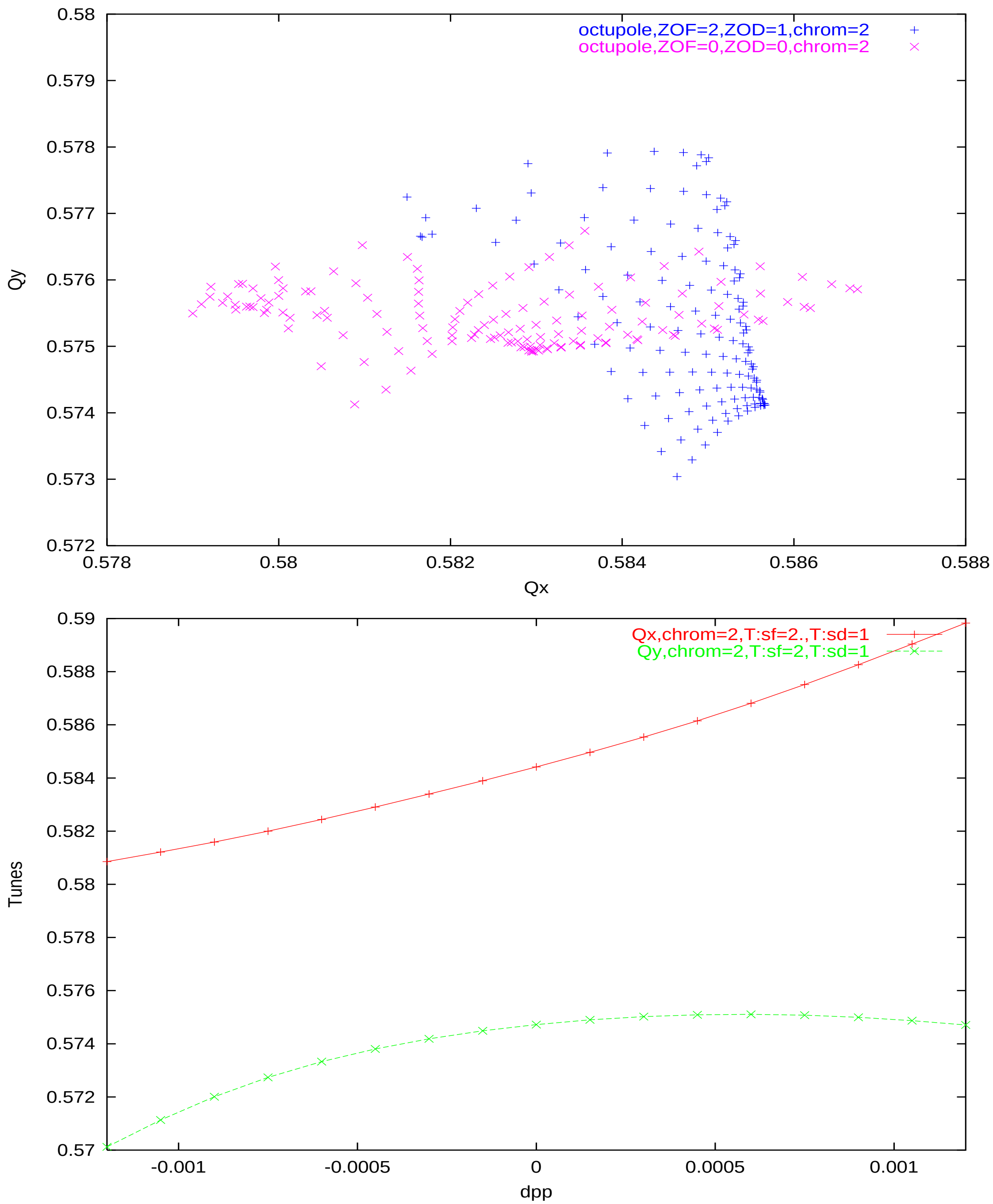


Figure 5: Tune spread due to nolinearities and chromaticities

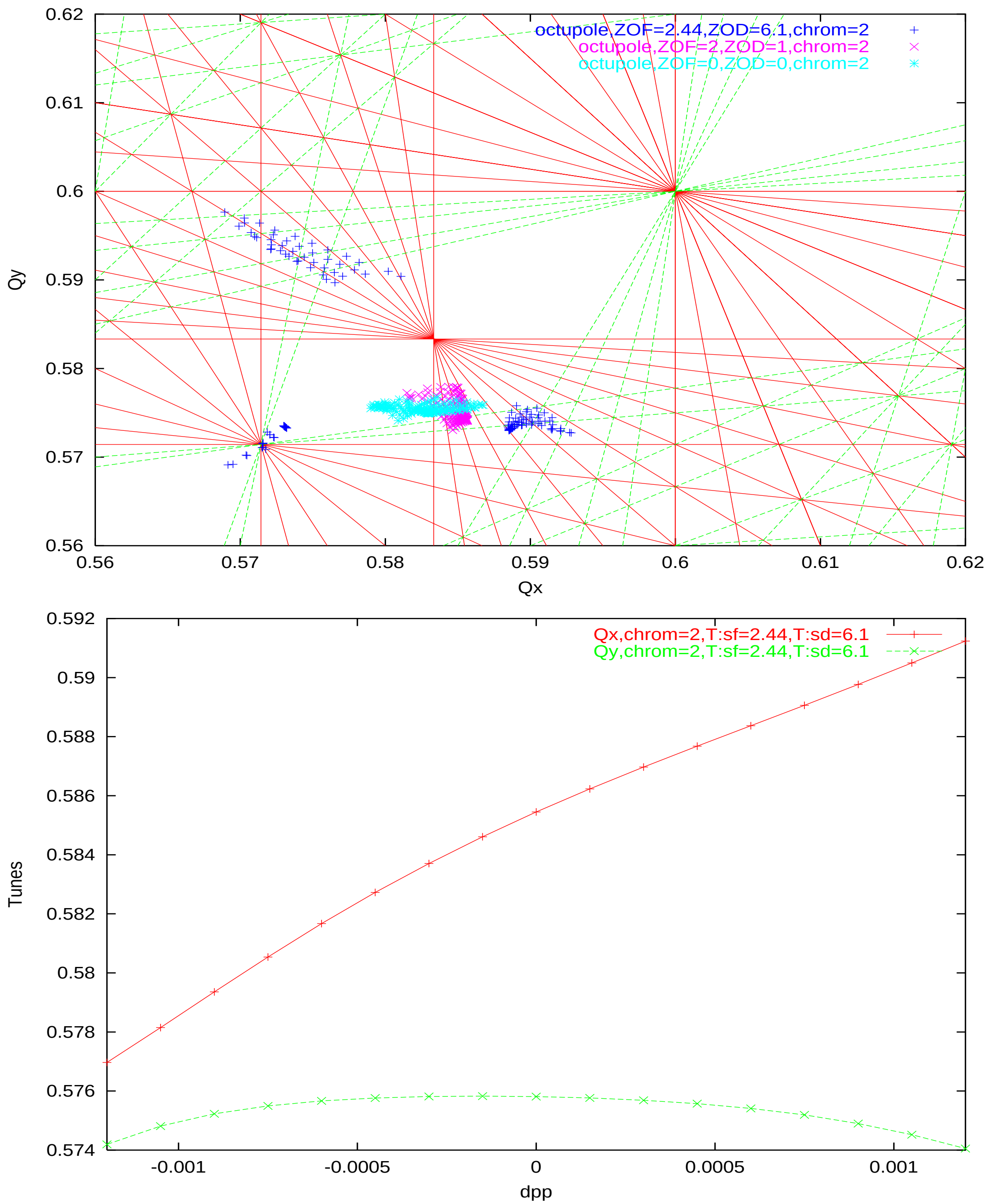


Figure 6: Tune spread with octupoles due to nolinearities and chromaticities